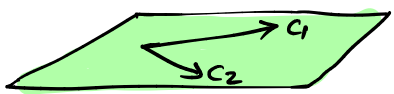
Linear Algebra

#2

$\rightarrow$ $A\bar{x} = \bar{b}$: when is this solvable? need to hold.

① $\bar{b}$ has to



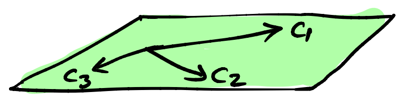
$\Downarrow$

- No way to
- Any $A\bar{x}$

② A has to be

$\Downarrow$

There shouldn't be of getting to $\bar{b}$ from columns of A.



$\Downarrow$

- Matrix is
- Matrix is
- Determinant (A)

$\rightarrow$ Rank(A) = No. of

$$\begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix} \Rightarrow$$ linearly if $\Rightarrow$

If dependent, then

$$\underbrace{w_1 c_1 + w_2 c_2}\ =$$

means $\bar{c}_3$ can be expressed as

- Null space : $\{ \bar{x} : \qquad \} \equiv$ space of all
  $N(A)$

- shape of null space :

$$A \rightarrow \begin{array}{c c} & n \\ m & \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\downarrow \qquad \downarrow$$

$$A^T \rightarrow \begin{array}{c c} & m \\ n & \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \end{bmatrix} \end{array} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\downarrow \qquad \downarrow$$

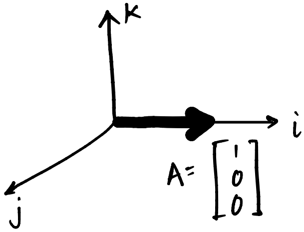
Observe that :

$\overline{c_{11} \; c_{21} \; c_{31}} \cdot \overline{y_1 \; y_2 \; y_3}$

$\overline{c_{12} \; c_{22} \; c_{32}} \cdot \overline{y_1 \; y_2 \; y_3}$

$\therefore$ $N(A^T) \perp$ and $N(A) \perp$

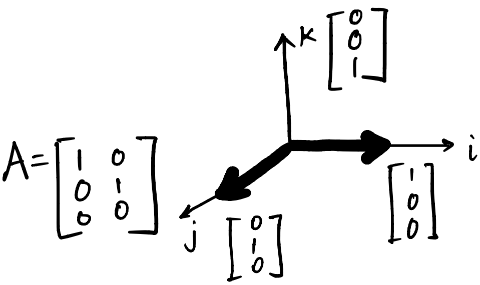
⊖ How large is $N(A^T)$?



T/F ?

Null space of this vector is vector $\bar{j}$ and $\bar{k}$

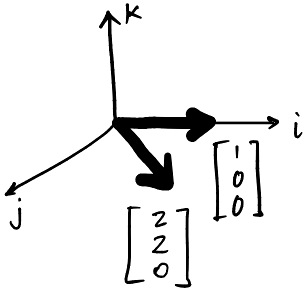
T/F : Null space of A^T contains $\bar{j}$ and $\bar{k}$



T/F ?

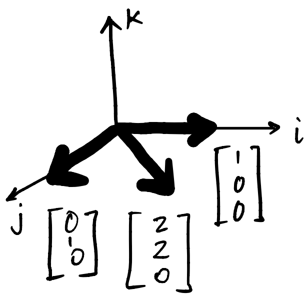
Null space of $A = \begin{bmatrix} | & | \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix} = \bar{k}$

- Null space of A^T contains $\bar{k}$



T/F ? N(A) does not exist

T/F ? $N(A^T)$ contains $\bar{k}$



T/F ? N(A) does not exist

T/F ? $N(A^T) = \phi$

$$A_{6\times 2} = \begin{bmatrix} 1 & 3 \\ 15 & 8 \\ 7 & 1 \\ 3 & 1 \\ 22 & 9 \\ 8 & 13 \end{bmatrix}$$

T/F ? $N(A) = \phi$

T/F ? $N(A^T) = \phi$

Now how many dimensions in $N(A^T)$?

$$A^T = \begin{bmatrix} 1 & 15 & 7 & 3 & 22 & 8 \\ 3 & 8 & 1 & 1 & 9 & 13 \end{bmatrix}$$

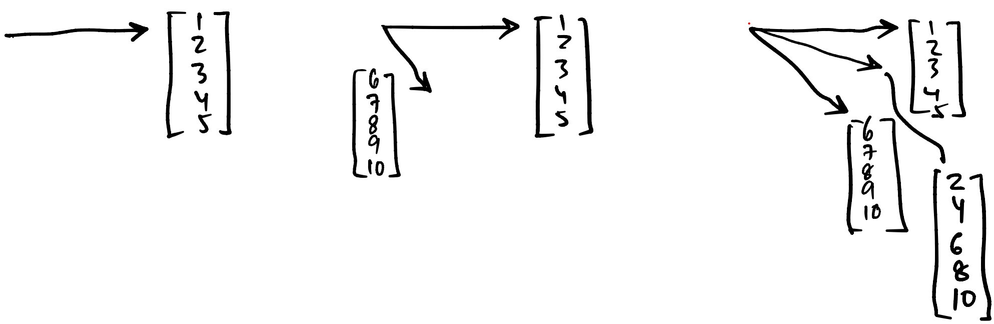
Think of this as

Generalize $(A_{m\times n})$

Intuition : Think of $N(A^T)$ as

Think of $N(A)$ as the

- Add m-dimensional cols. one by one to fill out as much space



Question: How many sol^n's possible to $A\bar{x} = \bar{b}$, and what dim. is $N(A)$?

(a)
$$\begin{array}{cc} & n \\ m & \begin{bmatrix} \text{Symm.} \\ \text{matrix} \end{bmatrix} \end{array}$$

Rank $= m = n$
Full rank

(b)
$$\begin{array}{cc} & n \\ m & \begin{bmatrix} \\ \text{Thin} \\ \text{matrix} \\ \\ \end{bmatrix} \end{array}$$

Rank $= n < m$
Full col. rank

(c)
$$\begin{array}{cc} & n \\ m & \begin{bmatrix} \quad\quad \text{fat matrix} \quad\quad \end{bmatrix} \end{array}$$

Rank $= m < n$
Full row rank

To see this, turn this matrix to a

(d)
$$\begin{bmatrix} \text{Matrix} \end{bmatrix}$$

Rank $< m$, Rank $< n$
Rank deficient matrix

- **Basis** : Linearly Independent
- **Dimensions** = = $|\text{Basis}|$
  - A space can have

Q: $\dim(C(A)) = ?$

- **Orthogonal vectors** :

Q: wall and floor orthogonal ?

- **Norm** : $m\begin{bmatrix} | \\ v_1 \\ | \end{bmatrix} \longrightarrow$ scalar $\Rightarrow \mathbb{R}^m \to \mathbb{R}$

- **Length** = L_2 norm =
- **L_0 norm** = # of
- **uv^T** =

- **Symmetric matrix** $\equiv$
- **$(AB)^{-1}$** = and $(AB)^T$ =
- **A^TA** is a