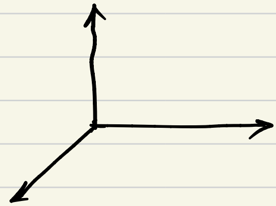
ECE/CS 434

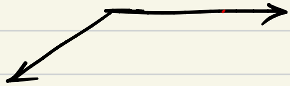
Linear Algebra Basics

ECE/CS 434 : Lin Alg. : Lecture 1

→ Vector Spaces :



Notion of space :



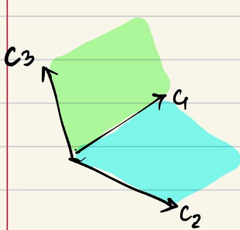
Possible to reach any point in the $\mathbb{R}^2$ space by

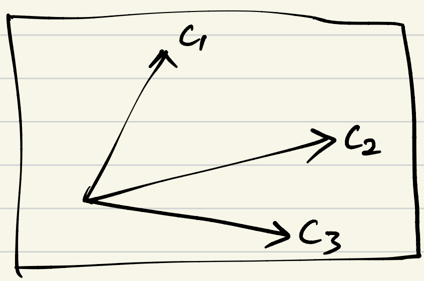
Informally : everything you can make by combining a given set of building blocks and rules.

→ Matrix $A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

→ Column space : All possible vectors that can be formed by

$$A\bar{x} = \begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$





$A\bar{x} = \bar{b}$

$\bar{b} \in \mathbb{R}$

$C(A) \in \mathbb{R}$

$A\bar{x} = \bar{b} \in$ col. space is the union of

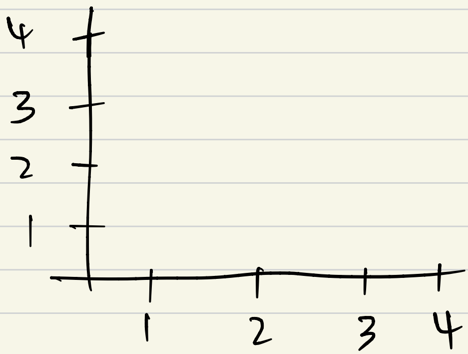
→ Row space : All possible vectors that can be formed by a weighted combination of A's

$$[x_1 \;\; x_2 \;\; x_3] \begin{bmatrix} \text{---} & r_1 & \text{---} \\ \text{---} & r_2 & \text{---} \\ \text{---} & r_3 & \text{---} \end{bmatrix} =$$

$=$

Example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \Rightarrow \begin{array}{l} r_1 = \\ r_2 = \end{array}$

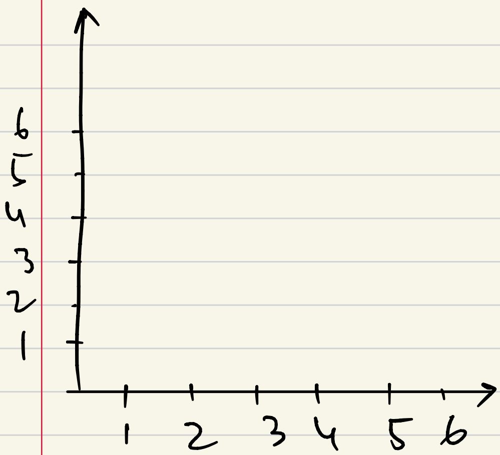
$\bar{x}^T A = \bar{b}^T \in$ row space of A = union of

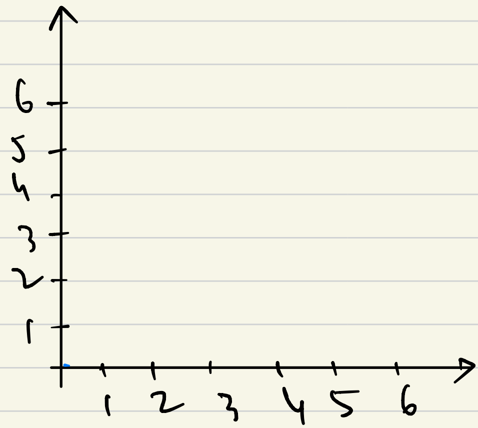


$x_1 r_1 + x_2 r_2 \in$

Question: Is R(A) and C(A) identical for

$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $\quad$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$





Vector space : set of all vectors W that satisfies 3 conditions

Question : space or not ?

(1) 2D plane ?

(2) One quadrant ?

(3) Line $x = y$?

(4) Zero vector ?

Popular matrix Equation : $A\bar{x} = b$

$$\begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

How do you solve this ?

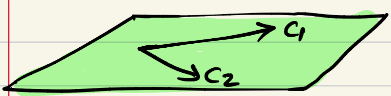
- Somehow make A
  - Get to a form like

- Gauss-Jordan : $\bar{x} = A^{-1}\bar{b}$
  - How do you get A^{-1} ?

$$\left[\begin{array}{c|ccc} A & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right]$$

$A\bar{x} = \bar{b}$: when is this solvable ? need to hold.

① $\bar{b}$ has to



⇓

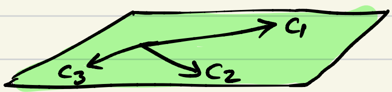
- No way to
- Any $A\bar{x}$

② A has to be

⇓

There shouldn't be

of getting to $\bar{b}$ from columns of A.



⇓

- Matrix is
- Matrix is
- Determinant (A)

Rank(A) = No. of

$$\begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix} \Rightarrow$$ Linearly if ⇒

If dependent, then

$$\underbrace{w_1 c_1 + w_2 c_2}$$ =

means $\bar{c}_3$ can be expressed as

- Null space : $\{\bar{x} : \quad\quad\quad \} \equiv$ space of all
  $N(A)$

- shape of null space :

$$\begin{array}{cc} & n \\ m & \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{array}{cc} & m \\ n & \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \end{bmatrix} \end{array} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\downarrow \qquad \downarrow \qquad\qquad \downarrow \qquad \downarrow$

Observe that :

$\overline{c_{11} \quad c_{21} \quad c_{31}} \cdot \overline{y_1 \quad y_2 \quad y_3}$

$\overline{c_{12} \quad c_{22} \quad c_{32}} \cdot \overline{y_1 \quad y_2 \quad y_3}$

$\therefore \quad N(A^T) \perp \qquad\qquad$ and $\quad N(A) \perp$