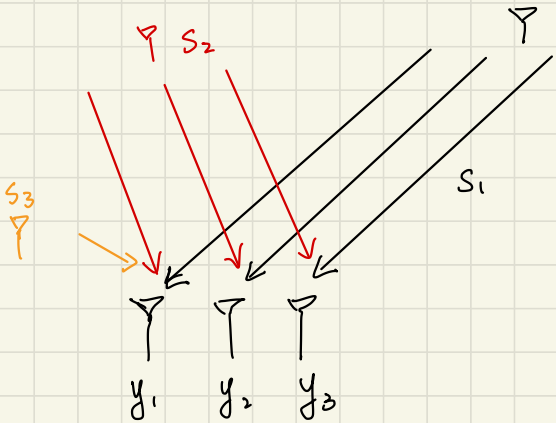
MVDR

Minimum Variance Distortionless Response



A algorithm

Problem :
Multiple sources at

Goal :
Maximize from
Suppress , at other angles

Analogy:
In the cocktail party, focus on Alice's speech

$$Y = AS + n = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_d \\ | & & | \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_d \end{bmatrix} + n \quad , \quad a_1 = \begin{bmatrix} \quad \end{bmatrix}$$

Apply to Y

$$w^* Y = \underbrace{\qquad}_{\text{Keep}} s_1 + \sum_{j=2}^{d} \underbrace{\qquad s_j +}_{\text{minimize}}$$

Formulation

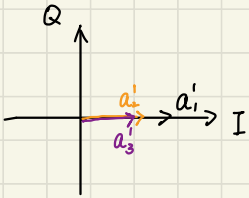
$\underset{w}{\text{argmin}}$ s.t

Example of 3 Tx, 4 mics

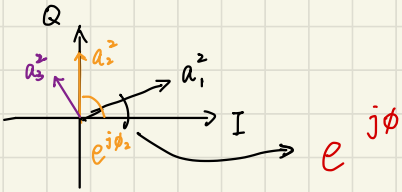
$$\begin{bmatrix} y^1 \\ y^2 \\ y^3 \\ y^4 \end{bmatrix} = \begin{bmatrix} a_1^1 & a_2^1 & a_3^1 \\ \vdots & \vdots & \vdots \\ a_1^4 & a_2^4 & a_3^4 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

$\hookrightarrow$

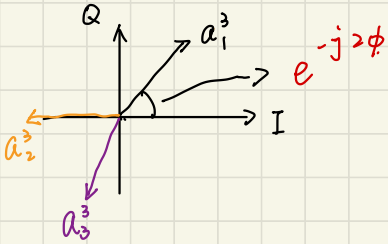
$y^1 =$



$y^2 =$



$y^3 =$



Q : How could we combine signals from 3 antennas

s.t. ① s_1 is kept

② s_2 s_3 are suppressed

$w_1 y_1 + w_2 y_2 + w_3 y_3$

$=$

are small

Formulation :

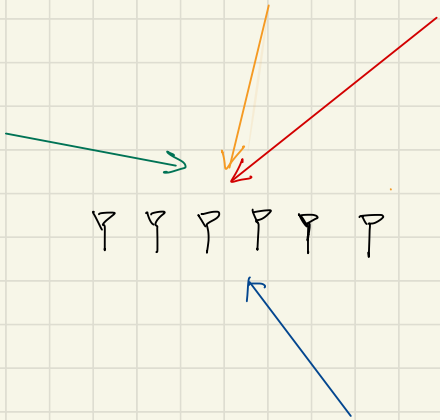
$$\underset{w}{\arg\min} \; \|w^* Y Y^* w\| \quad \text{s.t.} \; \|w^* a_1\| = 1$$

Solution w =

$w^* a_1$ = $\qquad\qquad\qquad\qquad$ = 1

MUSIC algorithm — MUltiple SIgnal Classification

Problem : Find θ_k for $k = 4$ Txs w/ 6 mics

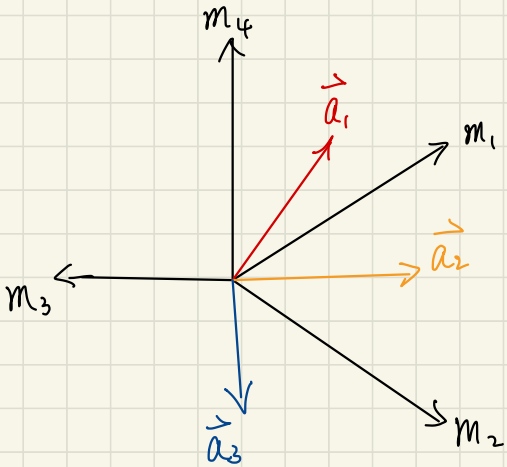


An algorithm

⇒ Delay and sum does not work well for

Why?

Microphone received space (complex, but here we draw real)



If are correlated

is large

Idea : Find the " " space

Lack color

Sub-space based AoA : MUSIC algorithm

$$Y = A\bar{s} + \bar{n}$$

$$YY^H = (As + n)(As + n)^H$$

$$= (As + n)(s^H A^H + n^H)$$

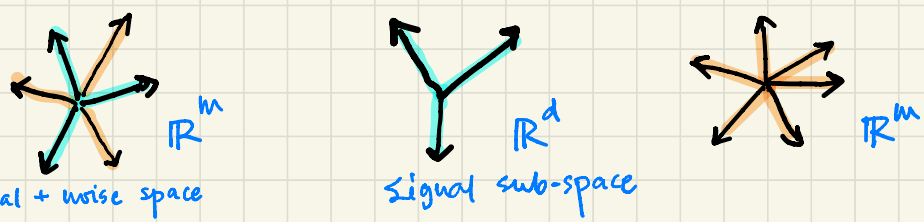
$$= Ass^H A^H + As.n^H + ns^H A^H + nn^H$$

$$E[YY^H] = E\left[Ass^H A^H + As.n^H + ns^H A^H + nn^H\right]$$

$$\underbrace{R_{YY}} = \underbrace{AR_{ss}A^H} + 0 + 0 + \underbrace{\sigma^2 I}$$

Intuition :

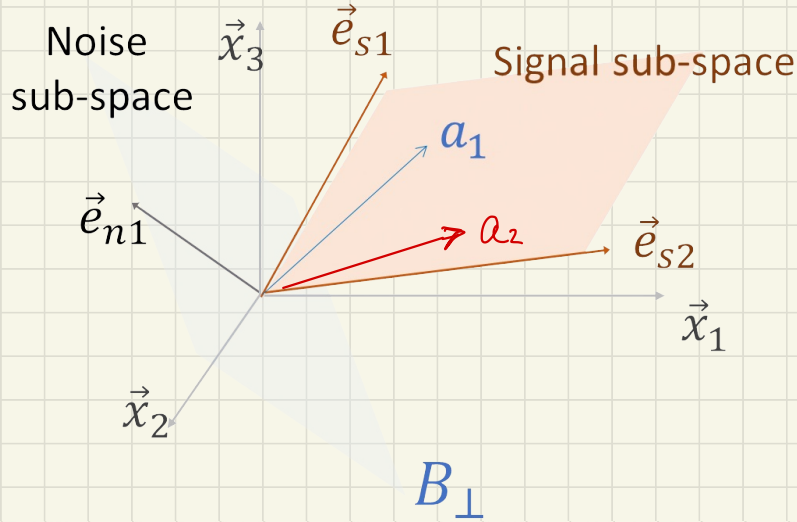
$$R_{xx} = AR_{ss}A^H + \sigma^2 I$$



$$(R_{xx} - \sigma^2 I)\,\bar{e}_{d+1} = 0 = AR_{ss}A^H . \bar{e}_{d+1}$$

Both full rank

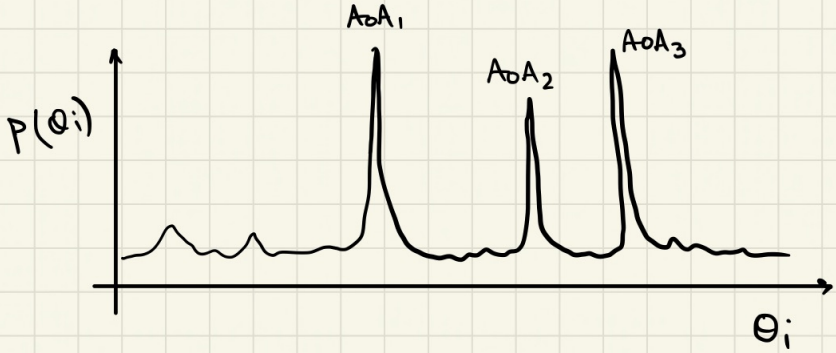
$$\therefore \quad A^H . \bar{e}_{d+1} = 0$$



AoA spectrum $P(\theta_i) = \dfrac{1}{\| A_{\theta_i}^H e_{noise} \|_{L_1}}$

$1 \times m$ $\quad$ $m \times (m-d)$

$$[-A_{\theta_i}-] \begin{bmatrix} | & & | \\ e_{d+1} & \cdots & e_m \\ | & & | \end{bmatrix} = [\cdots\cdots]$$



"When you have eliminated the impossible, whatever remains, however impropable, must be the truth" – Sherlock Holmes

Problem : When sources are " "

$$s_2 = \alpha s_1$$

$$Y = \begin{bmatrix} \quad \\ \\ \\ \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix} + n$$

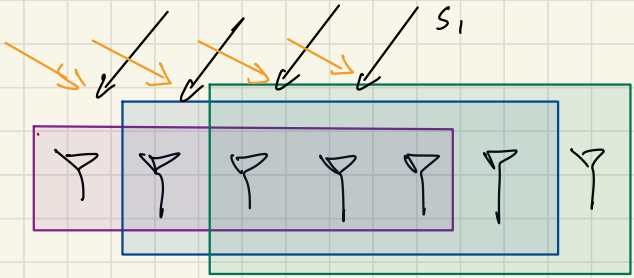
$$= \qquad\qquad + n$$

$$\begin{cases} 1. & \qquad\qquad (\text{rank } 1 < 2) \\ 2. & \vec{e}_{d+1} \perp \end{cases}$$

$P(\theta_2)$ may not be high !

MUSIC spatial smoothing

⇒ Separate the antenna array into



Note : $Y_1^2 = Y_2^1$

$$Y_1 = \begin{bmatrix} \quad \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} \quad \end{bmatrix}$$

$$+)\ Y_3 = \begin{bmatrix} \quad \end{bmatrix}$$

$$\sum_{g=1}^{G} Y_g = \begin{bmatrix} \quad \end{bmatrix}$$

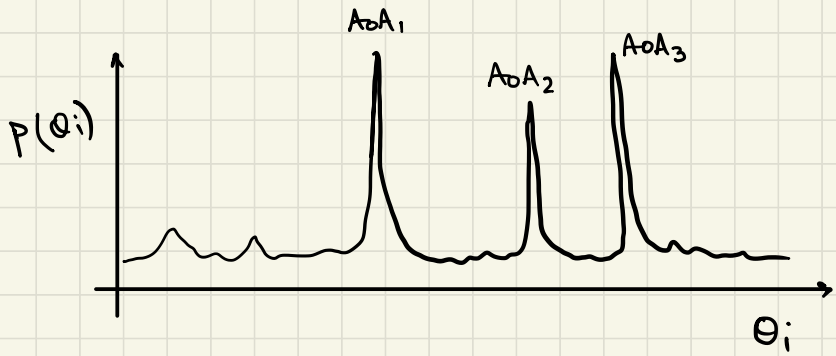
\+ Solve the correlated sources

\- Reduce the # of microphones in each group

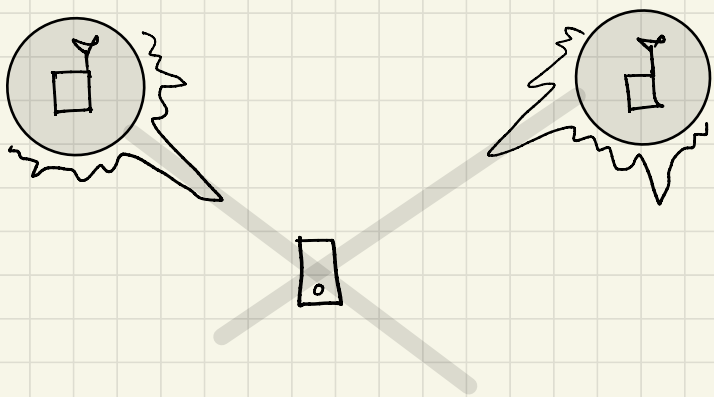
AoA spectrum $P(\theta_i) = \dfrac{1}{\| A_{\theta_i}^H e_{noise} \|_{L_1}}$

$1 \times m$ $\quad$ $m \times (m-d)$

$$\begin{bmatrix} - A_{\theta_i} - \end{bmatrix} \begin{bmatrix} | & & | \\ e_{d+1} & \cdots & e_m \\ | & & | \end{bmatrix} = \begin{bmatrix} \cdots & \cdots \end{bmatrix}$$



- WiFi :



- But what happens with multipath ?
  - LOS path varies slowly while multipath varies fast
    Identify LOS by observing stable AoA spectrum peak.
  - Apply Maximum Likelyhood Estimation (MLE) for localization