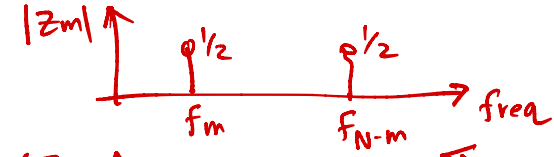
DSP : Discrete Fourier Transform (DFT)

3



Let's do 3 simple examples.

① $\mathrm{DFT}(\cos 2\pi f_m n t_s)$

② $\mathrm{DFT}(e^{j 2\pi f_m n t_s})$

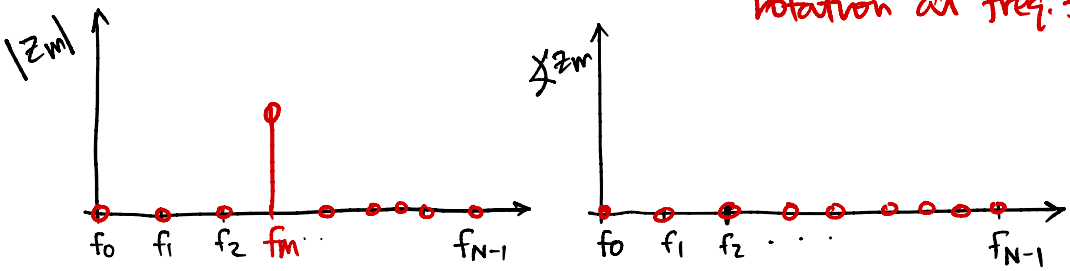
③ $\mathrm{DFT}(\sin 2\pi f_m n t_s)$

integer...

sampling period

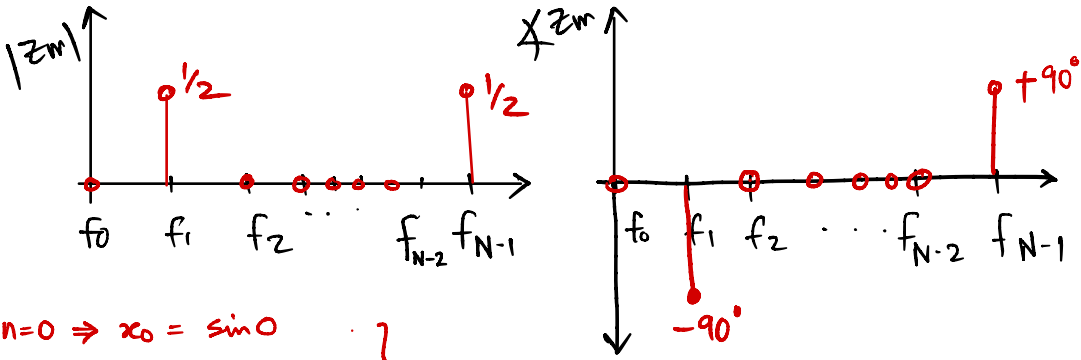
$$
\begin{aligned}
t_0 &\equiv x_0 = \\
t_1 &\equiv x_1 = \\
t_2 &\equiv x_2 = \\
&\vdots
\end{aligned}
\quad
\begin{bmatrix} e^{j0} \\ e^{j 2\pi f_m 1 t_s} \\ e^{j 2\pi f_m 2 t_s} \\ \vdots \end{bmatrix}
$$

Example 2: DFT ($x[n] = \underbrace{e^{j2\pi f_m n t_s}}_{\text{rotation at freq. } f_m}$)



Just one stick rotating at freq. f_m since it gives $e^{j\theta \cdot n}$

Example 3: DFT ($x[n] = \sin 2\pi f_1 n t_s$)



$n=0 \Rightarrow x_0 = \sin 0$

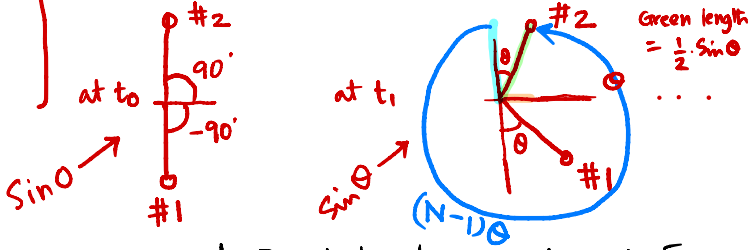
$n=1 \Rightarrow x_1 = \sin 2\pi f_1 t_s$

$n=2 \Rightarrow x_2 = \sin 2\pi f_1 2t_s$

$\vdots$

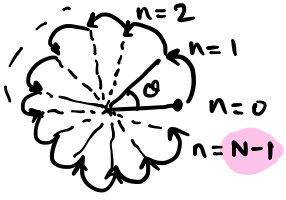
slow stick $\Rightarrow$ stick #1

fast stick $\Rightarrow$ stick #2



Like $\cos 2\pi f_1 n t_s$, you need 2 sticks to cancel out the complex values, but each stick now needs to start at different locations (or phases).

⊖ Translating to real-world frequencies



1 cycle in N time steps

1 cycle in $N \times \underline{t_s}$ seconds

x cycles in 1 second

$$\frac{1}{t_s} = \text{freq.} = f_s \qquad x = \frac{1}{N t_s} = \frac{1}{N} \cdot \frac{1}{t_s} = \frac{f_s}{N}$$

$\therefore$ slowest frequency $= \dfrac{f_s}{N}$

$\dfrac{f_s}{N} \Rightarrow$ Called Fundamental freq.

Faster freq. $\Rightarrow$ $2\dfrac{f_s}{N}, 3\dfrac{f_s}{N}, 4\dfrac{f_s}{N} \ldots (N-1)\dfrac{f_s}{N}$

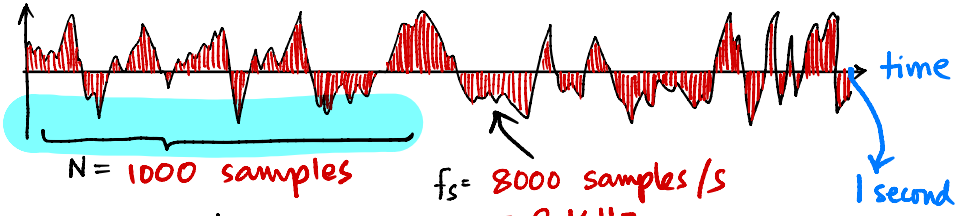
For large N, $(N-1) \approx N$,

$\therefore$ max freq $= f_s$

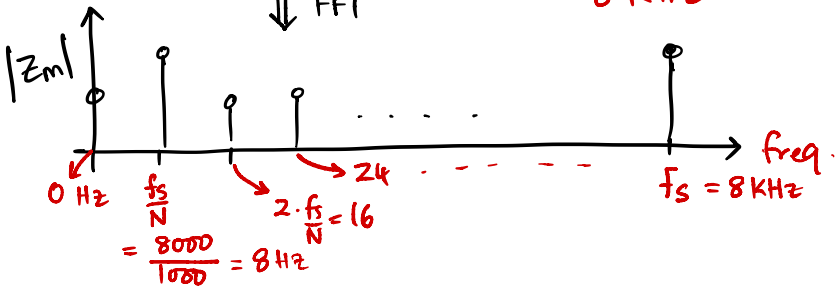
Bandwidth $= f_s$.

Let's put it all together

Consider sampling your voice signal at 8000 Hz and taking a 1000 point FFT (i.e., $f_s = 8000$ Hz, $N = 1000$)



$\Downarrow$ FFT



Thus, when analyzing a given signal, we have 2 knobs we can control

- Sampling freq f_s $\rightarrow$ Large f_s means we can see upto large freq components in the signal
- No. of samples you take in your FFT $= N$ $\rightarrow$ Large N means we have finer freq. resolution (or more freq. bins within $[0, f_s]$ band)

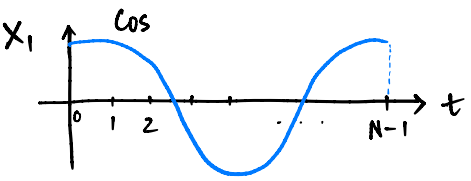
  Width of bin $= f_s/N$

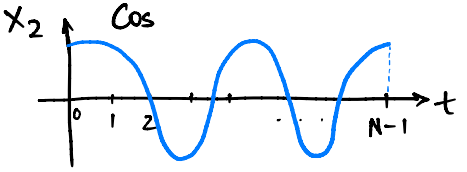
$f_1 = 1$

$\cos 2\pi f_2 n t_s$

$f_1 = 2$

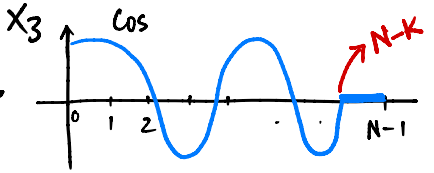
BEWARE OF INCOMPLETE CYCLES.



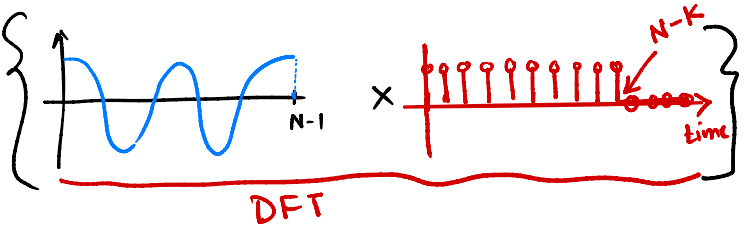


What is the FFT mag. spectrum for these signals?

Now, what is the FFT mag. spectrum of this →



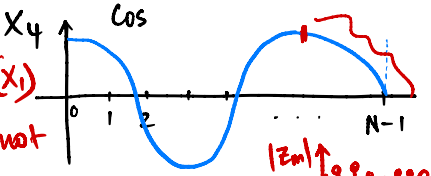
Think of this X_3 signal as →



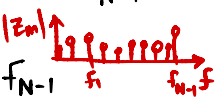
Thus, $DFT(X_3) \neq DFT(X_2)$ since the DFT ( ) influences results

In fact:

is $DFT(X_4) \neq DFT(X_2) \neq DFT(X_1)$

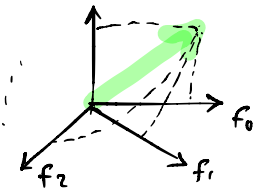


because 1¼ cos(·) cycles cannot be made up by integer no. of rotations of $\cos(f_1)$ & $\cos(f_{N-1})$



Many different rotations are needed ⇒

Think of this as

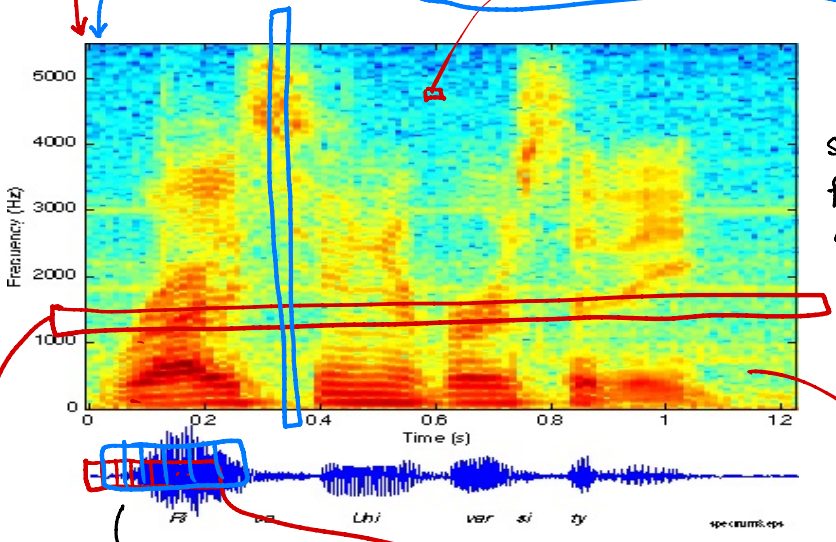


REAL SPECTROGRAM : Voice Signal

FFT of first window

FFT of sliding window

Time Frequency bin.



Shows various freq. components at this time slice.

Spectrogram

time domain signal

Shows how a specific freq. comes and goes over time.

$N = 1000$

$\Downarrow$

$FFT \in \mathbb{C}^{1000}$