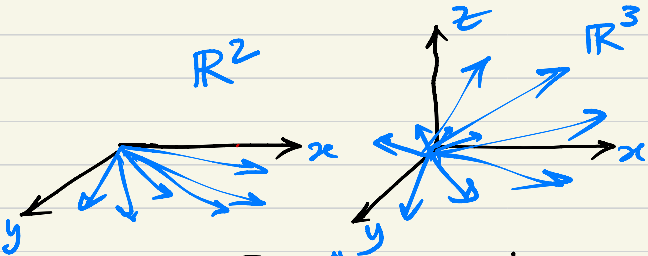
ECE/CS 434 : Lin Alg. : Lecture 1

→ Vector Spaces :

Notion of space :

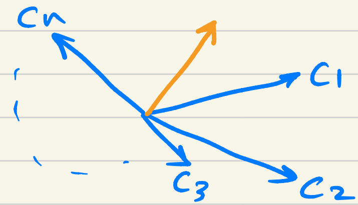


Possible to reach any point in the $\mathbb{R}^n$ space by combining N non-collinear vectors w/ apt. weights

Informally : everything you can make by combining a given set of building blocks and rules.

→ Matrix $A = \begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix}$

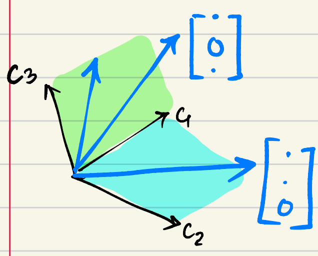
$\begin{bmatrix} 5 & 1 & 0 & 4 \\ 3 & 8 & 0 & 9 \\ 2 & 1 & 3 & 22 \end{bmatrix}$



→ Column space : All possible vectors that can be formed by a weighted combination of A's column vectors.

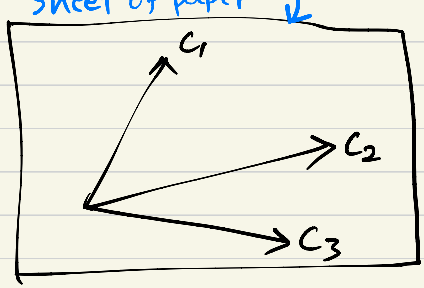
$C(A)$

$$A\bar{x} = \begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} | \\ c_1 \\ | \end{bmatrix} + x_2 \begin{bmatrix} | \\ c_2 \\ | \end{bmatrix} + \cdots x_3 \begin{bmatrix} | \\ c_3 \\ | \end{bmatrix}$$



$\bar{x}^T$

sheet of paper



$A\bar{x} = \bar{b}_c$

$\bar{b} \in \mathbb{R}^2$

$C(A) \in \mathbb{R}^2$

$[x_1 \ x_2 \ x_3] \begin{bmatrix} — r_1 — \\ — r_2 — \\ — r_3 — \end{bmatrix} = x^T A$

If $A\bar{x} = \bar{b}$, then col. space is the union of all possible $\bar{b}$ vectors

⊖ Row space : All possible vectors that can be formed by a weighted combination of A's row vectors

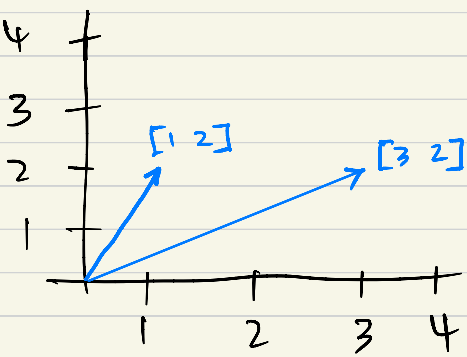
$$\underbrace{[x_1 \quad x_2 \quad x_3]}_{\text{Vector } \bar{x}^T} \underbrace{\begin{bmatrix} — r_1 — \\ — r_2 — \\ — r_3 — \end{bmatrix}}_{\text{Matrix } A} = x_1[— r_1 —] + x_2[— r_2 —] + x_3[— r_3 —] = \bar{b}_r^T$$

$=$

Example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \Rightarrow r_1 = [1 \quad 2]$, $r_2 = [3 \quad 2]$

$c_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $c_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\bar{x}^T A = \bar{b}^T \in$ row space of A = union of all feasible $\bar{b}^T$ vectors.

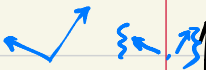


$x_1 r_1 + x_2 r_2 \in$

$R(A) \in \mathbb{R}^2$ ✓

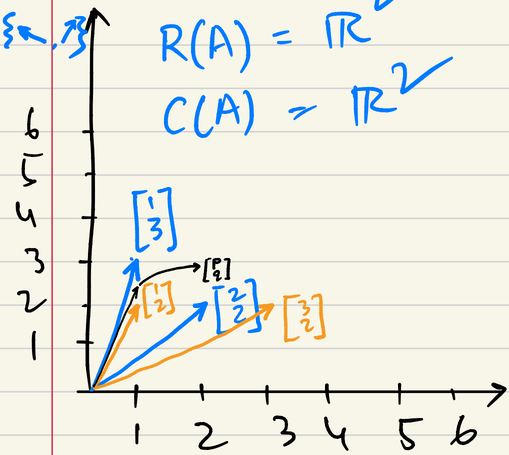
Question: Is R(A) and C(A) identical for

$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$



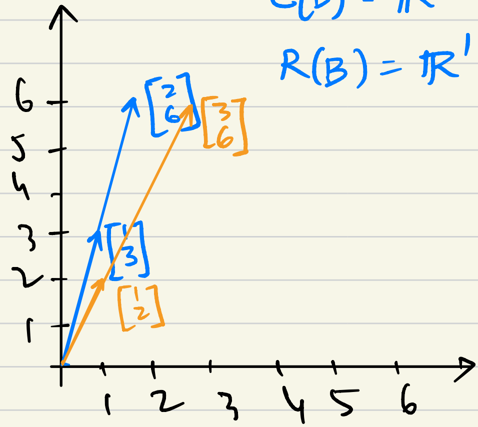
$R(A) = \mathbb{R}^2$ ✓

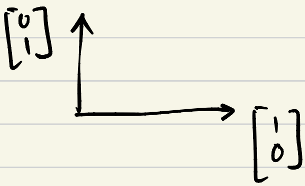
$C(A) = \mathbb{R}^2$ ✓



$C(B) = \mathbb{R}^1$

$R(B) = \mathbb{R}^1$





$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \forall x_1, x_2$$

➔ **Vector space** : set of all vectors W that satisfies 3 conditions

① if $\bar{u} \in W$, $\bar{v} \in W$, then $\bar{u} + \bar{v} \in W$

② if $\bar{u} \in W$, and $c.\bar{u} \in W$, c = constant

③ vector $\bar{0} \in W$.

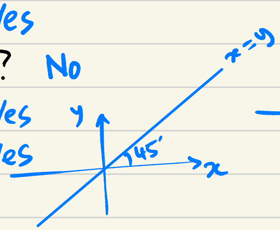
Question : space or not ?

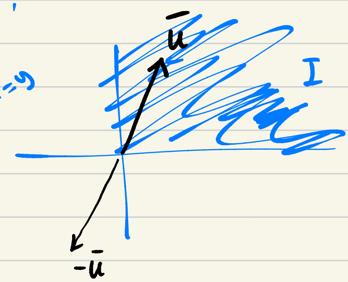
(1) 2D plane ? Yes

(2) One quadrant ? No

(3) Line $x = y$? Yes

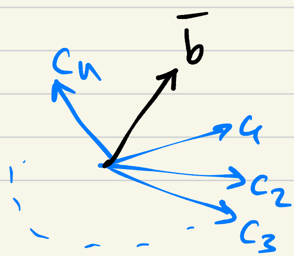
(4) Zero vector ? Yes





➔ Popular matrix Equation : $A\bar{x} = b$

$$\begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$



How do you solve this?

$$\begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix} \vdots \\ x \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ b \\ \vdots \end{bmatrix}$$

- Somehow make A
  - Get to a form like

$$\begin{bmatrix} & \\ 0 & \end{bmatrix}\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} b' \end{bmatrix}$$

- Gauss-Jordan : $\bar{x} = A^{-1}\bar{b}$
  - How do you get A^{-1}?

$$\left[\begin{array}{c|ccc} & 1 & 0 & 0 \\ A & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \cdot\ \cdot\ \cdot \\ 0 & 1 & 0 & \cdot\ \cdot\ \cdot \\ 0 & 0 & 1 & \underbrace{\cdot\ \cdot\ \cdot}_{A^{-1}} \end{array}\right]$$