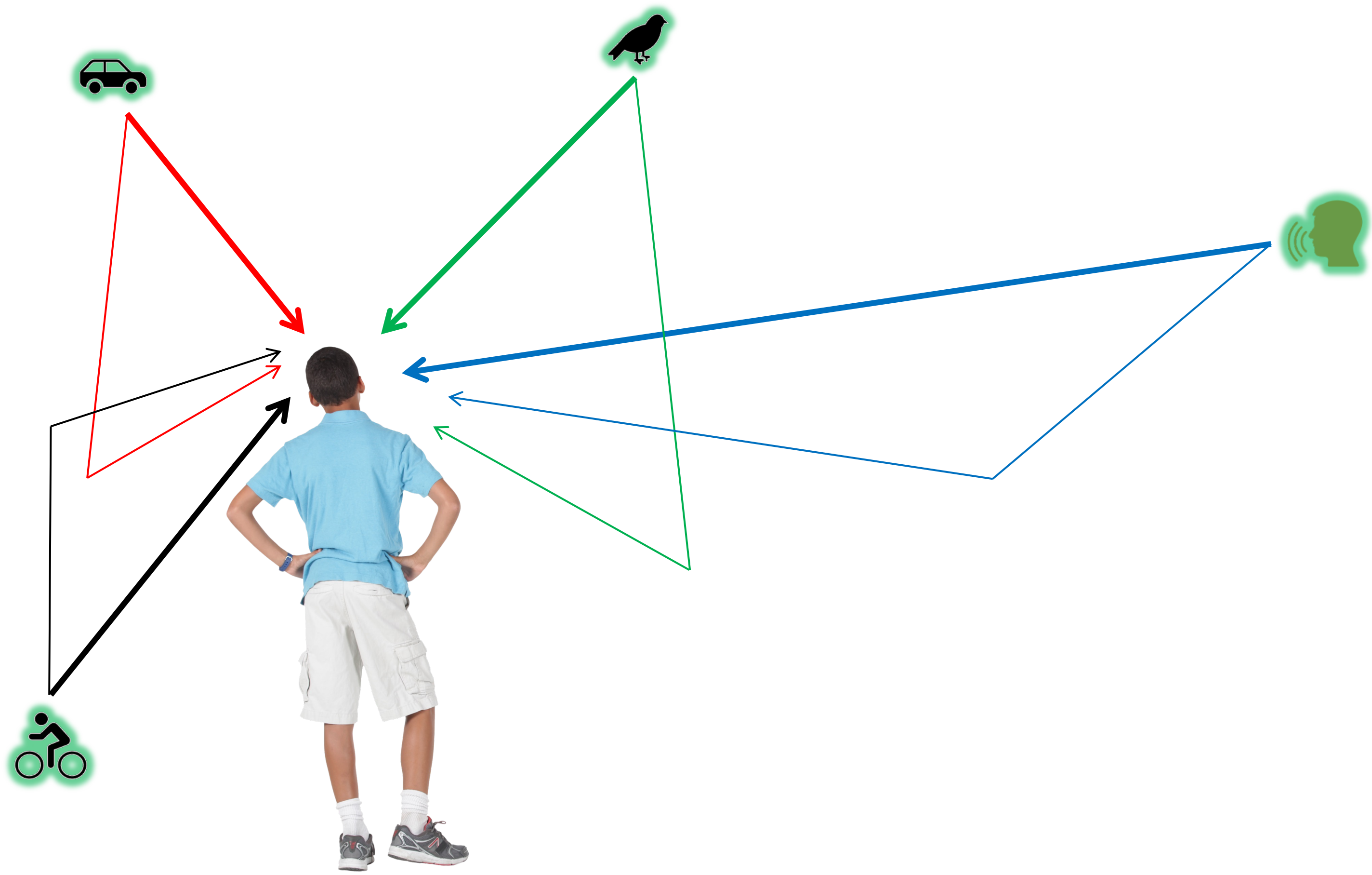
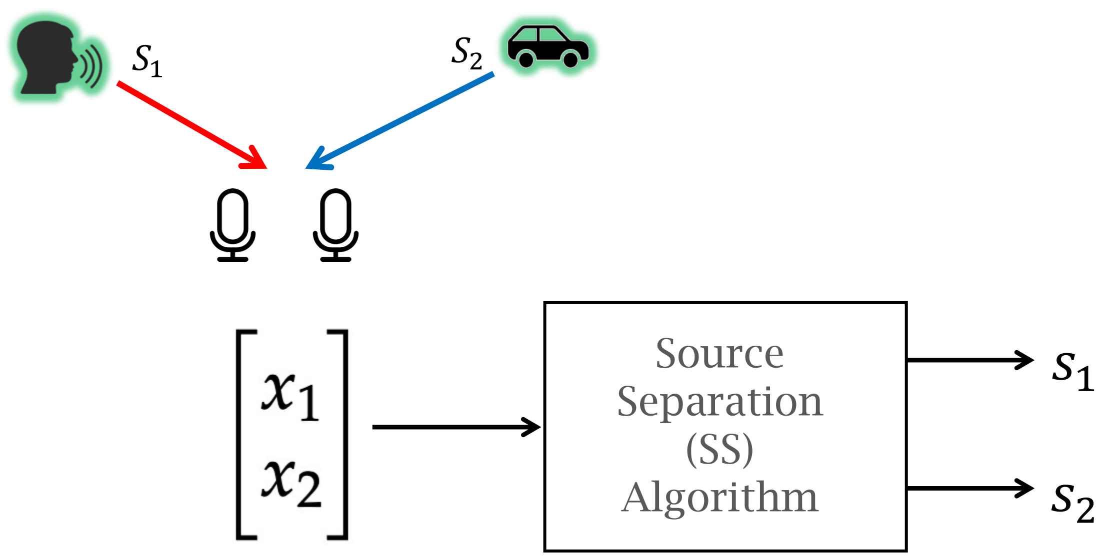
Source separation



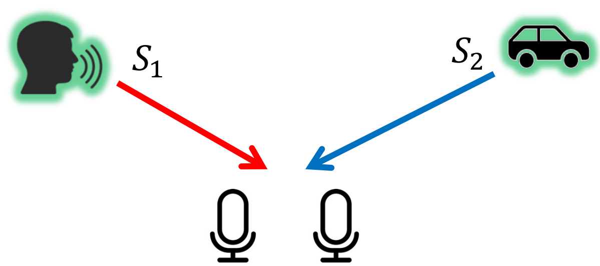
Source separation preliminaries

Source separation: The general problem statement



Source separation preliminaries

Source separation: The general problem statement

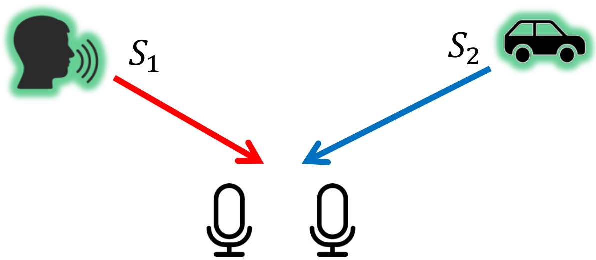


$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}}_{\text{Mixing matrix}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Unknown mixing matrix, unknown source signals → heavily under-determined

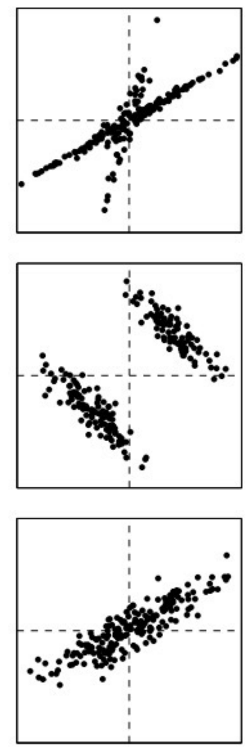
Source separation preliminaries

Source separation: The general problem statement



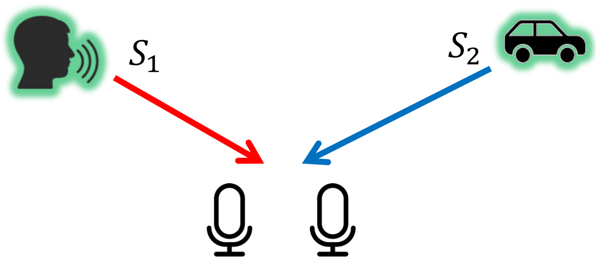
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}}_{\text{Mixing matrix}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$



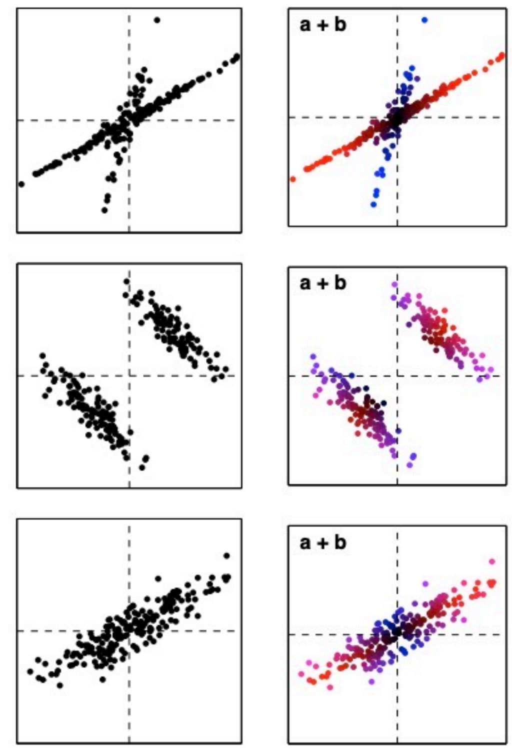
Source separation preliminaries

Source separation: The general problem statement



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}}_{\text{Mixing matrix}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

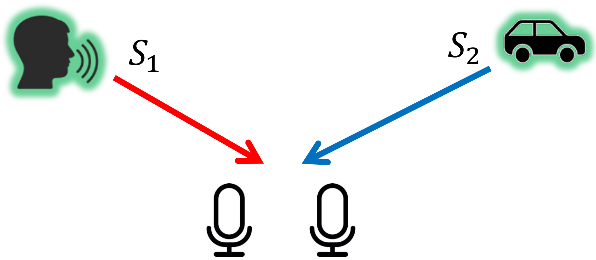
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$



Hard to separate the sources even visually

When can we solve SS?

Let’s make some simplifications: Mixing matrix is AoA (or steering) matrix

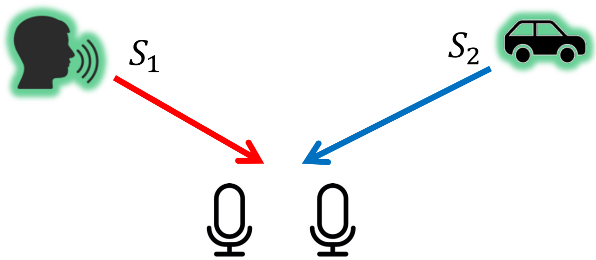


$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ \vec{a}_{\theta_1} & \vec{a}_{\theta_2} \\ | & | \end{bmatrix}}_{\text{Steering matrix A}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Steering matrix A

When can we solve SS?

Let’s make some simplifications



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ \vec{a}_{\theta_1} & \vec{a}_{\theta_2} \\ | & | \end{bmatrix}}_{\text{Steering matrix A}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Steering matrix A

$$X = A.S + N$$

1) Matrix A is known

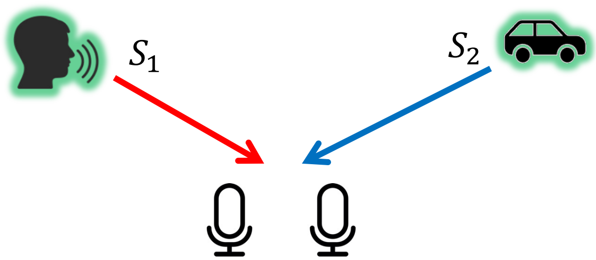
- Problem is determined, easy to solve
- With noise, apply Least Squares

When can we solve SS?

$$X = A.S + N$$

Let's make some simplifications

3) A and S both unknown, but s1 and s2 are independent

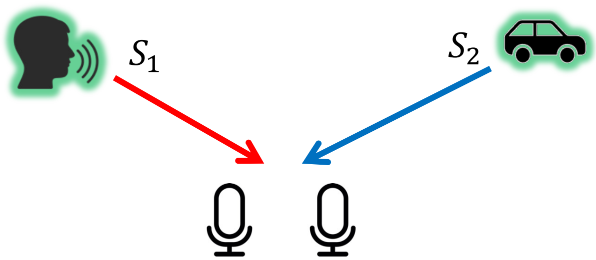


$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ \vec{a}_{\theta_1} & \vec{a}_{\theta_2} \\ | & | \end{bmatrix}}_{\text{Steering matrix A}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

When can we solve SS?

$$X = A.S + N$$

Let's make some simplifications



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ \vec{a}_{\theta_1} & \vec{a}_{\theta_2} \\ | & | \end{bmatrix}}_{\text{Steering matrix A}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Steering matrix A

3) A and S both unknown, but s1 and s2 are independent

- Independent component analysis (ICA)

Step 1 Model covariance of x as a function of A's SVD

$$E[xx^T] = (As)(As)^T = Ass^TA^T = (U\Sigma V^T)ss^T(V\Sigma^T U^T)$$
$$= U\Sigma^2 U^T$$

Step 2 Compute Covariance of x, match with model

$$E[xx^T] = EDE^T = U\Sigma^2 U^T$$

Note: We now know $U = E$ and $\Sigma = D^{1/2}$ is the only unknown

Step 3 Write out source estimate in terms of V

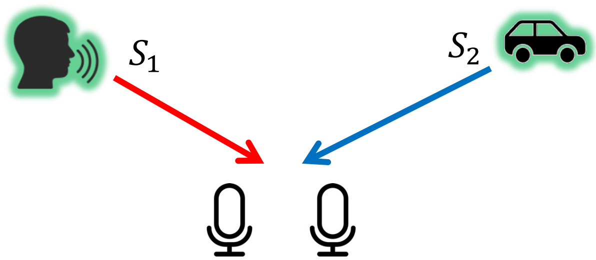
$$\hat{s} = A^{-1}x = (U\Sigma V^T)^{-1}x = (V\Sigma^{-1}U^T)x = (VD^{-1/2}E^T)x$$

Step 4 How can we find V? Exploit higher order statistics

$$V^* = \underset{V}{argmax}\ KL\_Div\left((VD^{-1/2}E^T)x\right)$$

When can we solve SS?

Let's make some simplifications



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} | & | \\ \vec{a}_{\theta_1} & \vec{a}_{\theta_2} \\ | & | \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Steering matrix A

$$X = A.S + N$$

3) A and S both unknown, but s1 and s2 are independent

- Independent component analysis (ICA)

Step 1 Model covariance of x as a function of A's SVD

$$E[xx^T] = (As)(As)^T = Ass^TA^T = (U\Sigma V^T)ss^T(V\Sigma^T U^T) = U\Sigma^2 U^T$$

Step 2 Compute Covariance of x. match with model

$$E[xx^T] = EDE^T = U\Sigma^2U^T$$

Note: We now know $U = E$ and $\Sigma = D^{1/2}$ V is the only unknown

Step 3 Write out source estimate in terms of V

$$\hat{s} = A^{-1}x = (U\Sigma V^T)^{-1}x = (V\Sigma^{-1}U^T)x = (VD^{-1/2}E^T)x$$

Step 4 How can we find V? Exploit higher order statistics

$$V^* = \underset{V}{argmax}\ KL\_Div\left((VD^{-1/2}E^T)x\right)$$

4) Independent vector analysis (IVA)

5) Matrix factorization methods

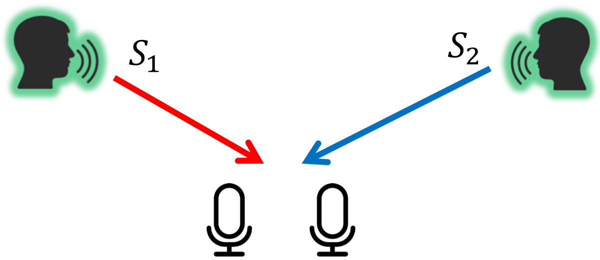
6) Matrix completion methods

Source = Speech

$$X = A.S + N$$

Let's make some simplifications

When the source signal is speech …

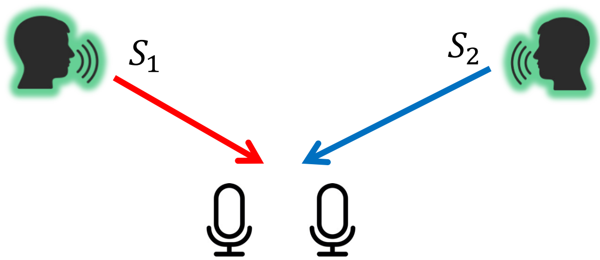


$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ \vec{a}_{\theta_1} & \vec{a}_{\theta_2} \\ | & | \end{bmatrix}}_{\text{Steering matrix A}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Source = Speech

$$X = A.S + N$$

Let's make some simplifications



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \vec{a}_{\theta_1} & \vec{a}_{\theta_2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Steering matrix A

When the source signal is speech, exploit TF-disjointness