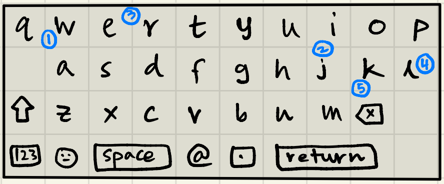
Some other applications of HMM (informal discussion)

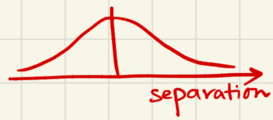
I Auto-correction in smartphone keyboard.



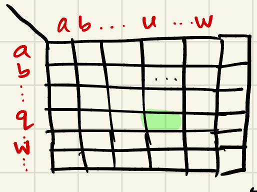
wield
quell
qualm
} hide

$$P(m_1 | s_1) = P\left(m_1 = \text{location of } 1 \;\middle|\; s_1 = \begin{bmatrix} a \\ b \\ c \\ d \\ \vdots \\ z \end{bmatrix}\right) =$$

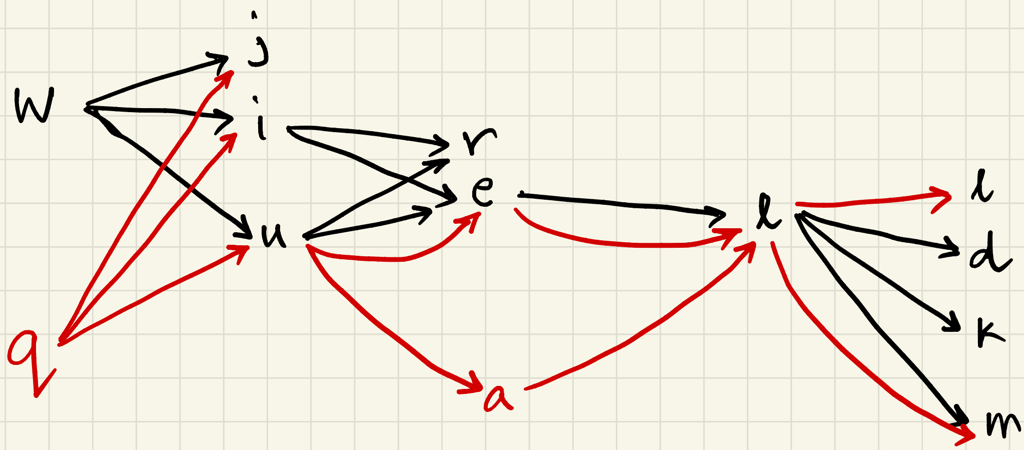
2D Gaussian



$$P(s_2 | s_1) = P\left(s_2 = \begin{bmatrix} j \\ i \\ u \\ h \\ o \\ k \\ \vdots \end{bmatrix} \;\middle|\; s_1 = \begin{bmatrix} w \\ q \\ r \\ a \\ e \\ s \\ \vdots \end{bmatrix}\right) =$$



} from English dictionary

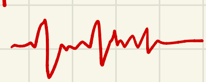


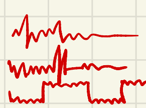
$\Downarrow$

Decodes to "quell" or "qualm"

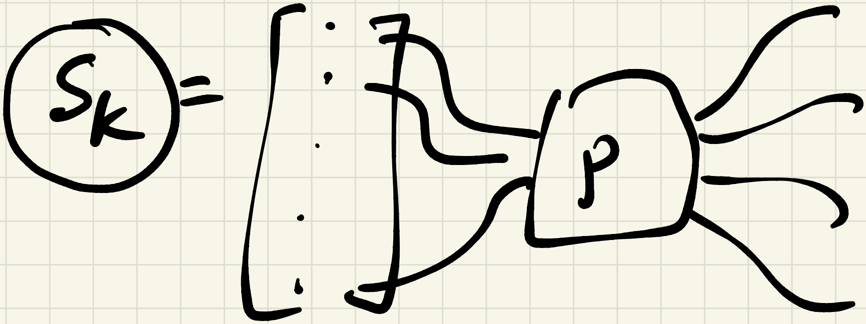
Similar application in speech recognition

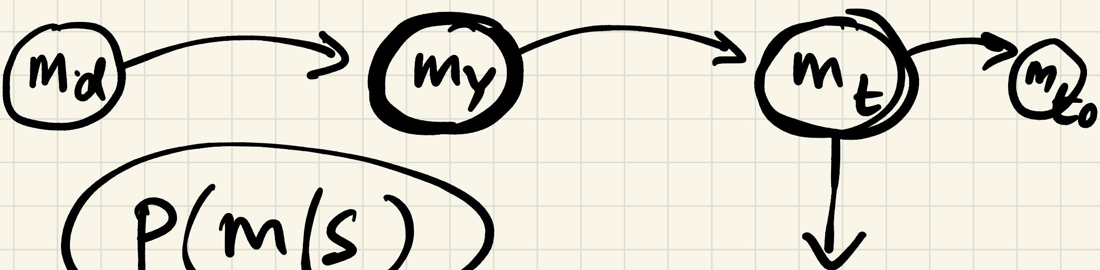
$$P(m_K | s_K) = P\left(m_K = \text{[waveform]} \;\middle|\; s_K = \begin{bmatrix} a' = \text{[waveform]} \\ b' = \text{[waveform]} \\ K' = \text{[waveform]} \\ \vdots \end{bmatrix}\right)$$





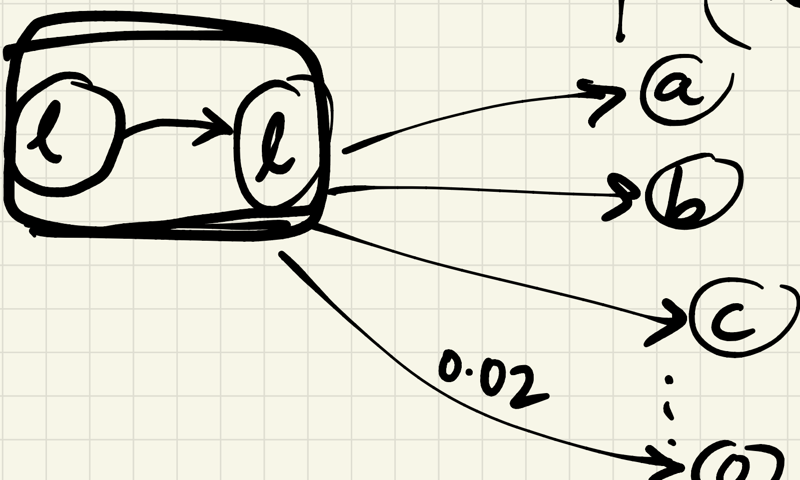
$S_K \Rightarrow$ Random Var.





$P(m|s)$

$P(l \longrightarrow o) = 0.02$

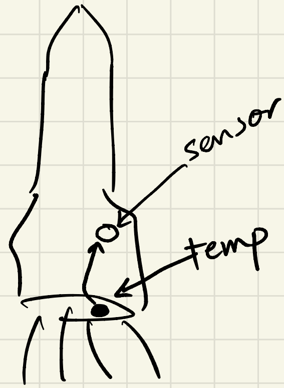


0.14

$\downarrow$

ll $\underline{0}$

Kalman Filter



temp. inside the jet x_k

$$y_k = \underline{\underline{H}}\, x_k + n_m$$

measurement eq^n

gradient of temp on the body of the rocket.

$$x_{k+1} = A x_k + B + n_p$$

process model eq.

Goal : Estime x_k or track x_k

$\hat{x}_k \equiv$ my estimate of x_k

Main KF equation says : combine both process and measurement

$$\hat{x}_k = x_k^p + G\left(y_k - H x_k^p\right)$$

$$\hat{x}_k = x_k^p + G\left(H x_k + n_m - H x_k^p\right)$$

$$= x_k^p + GH\left(x_k - x_k^p\right) + G n_m$$

net error $e_k = x_k - \hat{x}_k$

$$e_k = x_k - \left(x_k^p + GH\left(x_k - x_k^p\right) + Gn_m\right)$$

$$= x_k\left(1 - GH\right) - \left(1 - GH\right)x_k^p - Gn_m$$

$$e_k = \left(1 - G_k H\right)\left(x_k - x_k^p\right) - G_k n_m$$

$e_k^p \equiv$ process error

$$\text{Covariance}\left(e_k\right) = E\left[e_k e_k^T\right]$$

$$= E\left[\left\{\left(1 - G_k H\right)e_k^p - G_k n_m\right\}\left\{\cdots\right\}^T\right]$$

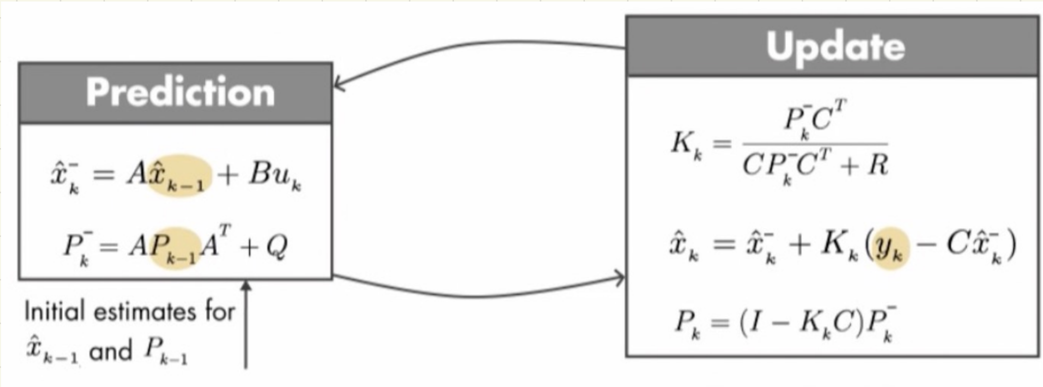
to find G_k that minimizes the error covariance,

$$\frac{d\,\mathrm{Trace}\left(E\left[e_k e_k^T\right]\right)}{dG_k} = 0$$

You get $G_k^* = \dfrac{P_k^p H^T}{H P_k^p H^T + R_m}$

where $P_k^p = E\left[e_k^p \left(e_k^p\right)^T\right]$

$$\mathrm{Cov}\left(\begin{bmatrix} e_k \\ e_{k+1} \\ e_{k+2} \\ \vdots \end{bmatrix}\right) = \begin{bmatrix} e_k e_k^T & & \\ & e_{k+1} e_{k+1}^T & \\ & & e_{k+2} e_{k+2}^T \end{bmatrix}$$



$$Q = E[n_p n_p^T]$$

$$R = E[n_m n_m^T]$$