GCC - PHAT General Cross Correlation with PHAse Transform

An AoA detection algorithm

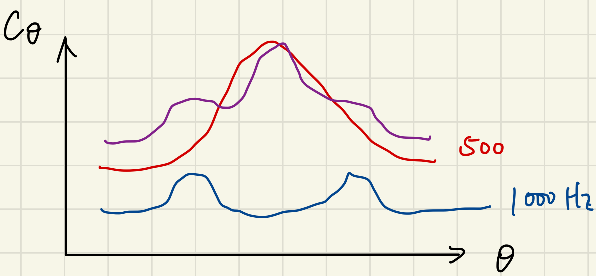
$$C_{\theta_i} = [-\ a_i\ -]^* \begin{bmatrix} | & | & | \\ a_1 & a_i & a_d \\ | & | & | \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_d \end{bmatrix} + [-\ a_i\ -]^* \begin{bmatrix} n_0 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

$$= \|a_i\|^2 s_i + \sum_{k=1}^{d} a_i a_j s_j + a_i^* n$$

$\Rightarrow$ $C_{\theta_i}(f)$ is proportional to s_i / s_j (amplitude on each frequency)

$\theta_i(f_1) \approx \theta_i(f_2)$

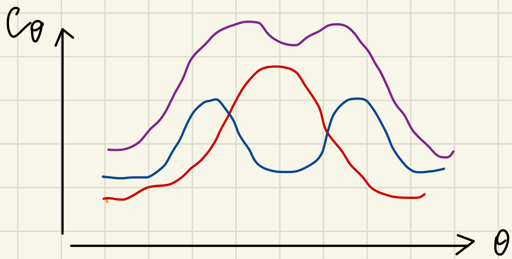
Q: How to combine $C_{\theta_i}(500\text{Hz})$ and $C_{\theta_i}(1000\text{ Hz})$



$$\sum_f C_{\theta_i}(f)$$

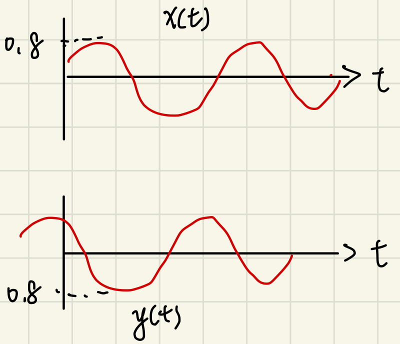
$\Rightarrow$ Dominate by strong frequencies

Equalize each frequency w/ amplitude



$$\text{GCC-PHAT}(\underset{a_i^*}{x}, y, f) = \frac{\overbrace{X(f)^* Y(f)}^{\text{GCC}}}{\underbrace{\|X(f)^* Y(f)\|}_{\text{PHAT} \to \text{Equalize amplitude}}}$$

e.g.



$$X(f) = 0.8\, e^{j\frac{\pi}{4}}$$

$$Y(f) = 0.8\, e^{j\frac{5\pi}{4}}$$

$$GCC(x, y, f) = \frac{0.64\, e^{j\pi}}{0.64} = -1 = e^{j\pi}$$

⇒ Take only phase per frequency

Ignore the amplitude

Advantage :

- Robust against correlated sources (echo)

  $s_2 = s_1 + \alpha s_3$

  where $C_{\theta_i} = \|a_i\|^2 s_i + \sum_{k=1}^{d} \underbrace{a_i^* a_j s_j}_{\text{Large}}$

  $E[a_i^* a_j] = 0$

Disadvantage :

- Low power / noisy frequecies pollutes the estimate