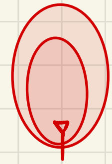
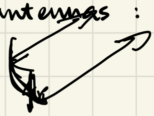
Beamforming and Angle of Arrival (AoA)

- Omnidirectional antennas: radiate signals equally in all directions

Directional antennas : Direct the radiation more in certain directions and less in others.

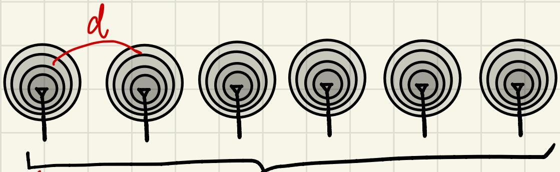




- creating such non-circular radiation patterns ⇒ Beamforming → Spatial Filter

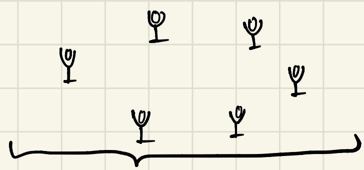
How ?

- Let's consider an ARRAY of omni-directional antennas (or even microphones)



omni directional

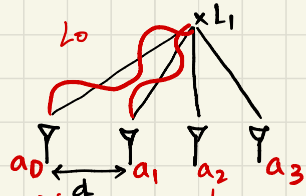
Linear antenna array



Circular mic. array (Alexa)

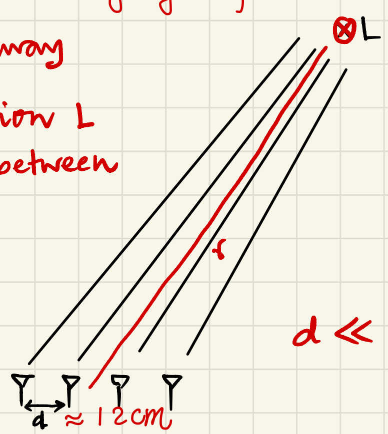
- say, these antennas transmit all at the same time?
  - ↳ What signals will you receive from different locations?

- consider nearby locations first :
  - ↳ The aggregate signals at these nearby locations vary based on the location.
  - ↳ No pattern is visible as you move.
  - ↳ This is called " NEAR FIELD ".



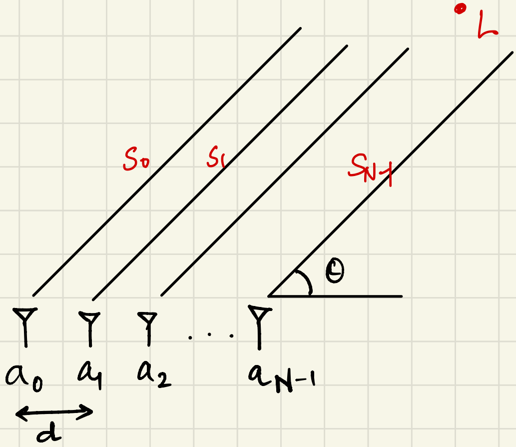
L_0 ≫ slightly larger than d

- Now, consider locations that are far away
  - ↳ When distance from antennas to location L becomes ≫ than seperation 'd' between the antennas, then the signal paths almost become PARALLEL
  - ↳ called "FAR FIELD"
  - ↳ Let's analyze far field effects



$d \ll r$

③



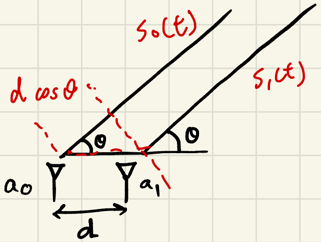
- All antennas transmit
- Say Rx receives $s_0(t)$ from antenna a_0 ... and $s_i(t)$ from antenna a_i
- Received signal $y(t)$

$$y(t) = \sum_{k=0}^{N-1} S_k$$

⑤ Now, assume direct path (no echo or multipath).

↳ Then what is the difference between $s_0(t)$ and $s_1(t)$?

Ans :



$s_1(t)$ travels $d\cos\theta$ less distance than $s_0(t)$.

⇓

How much phase shift ϕ does this cause? $\phi = \frac{2\pi}{\lambda} d\cos\theta$

500Hz

acoustic sounds

λ examples

$V_c = 340 m/s$

$V_c = \lambda f$

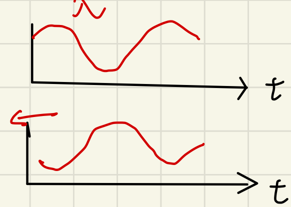
↳ 500Hz

$\lambda = 68$ cm

λ distance causes 2π phase shift

∴ $d\cos\theta$ causes $\frac{2\pi}{\lambda} d\cos\theta$ phase shift

E.g.



180° phase shift

⇓

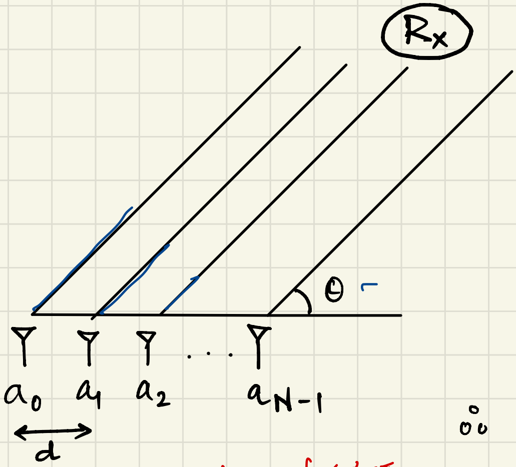
How can we mathematically write that $s_1(t)$ = $s_0(t)$ phase shifted by $\phi = \frac{2\pi}{\lambda} d\cos\theta$

↳ Recall phase shift ≡ time shift the signal

Thus :

$s_0(t) = \cos(2\pi f_1 t)$ $\quad s_1(t) = \cos(2\pi f_1 t + \phi)$

$$\therefore \; s_1(f) = S_0(f)\, e^{j\phi}$$



$$s_1 = s_0 e^{j\phi}, \quad \phi = \frac{2\pi}{\lambda} d\cos\theta$$

$$s_2 = s_0 e^{j2\phi}$$

$$\vdots$$

$$s_{N-1} = s_0 e^{j(N-1)\phi}$$

$$\therefore \|Y\| = \sum_{k=0}^{N-1} s_k = \sum_{k=0}^{N-1} s_0 e^{jk\phi}$$

$$\cong s_0\left(\frac{1 - e^{jN\phi}}{1 - e^{j\phi}}\right), \quad \phi = \frac{2\pi}{\lambda} d\cos\theta$$

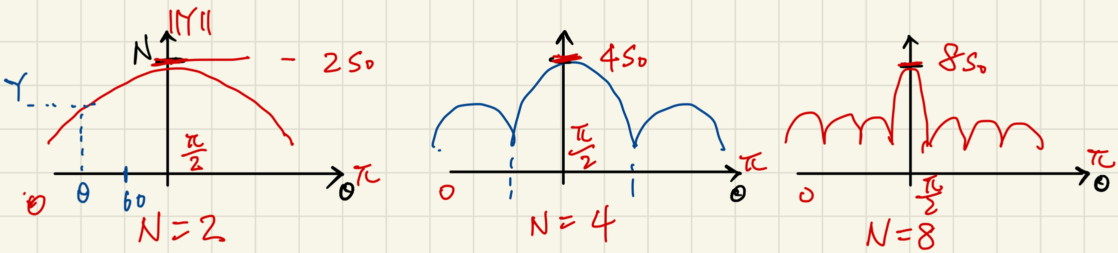
cancel example, if $4\phi = \pi$

$$s_4 = s_0 \boxed{e^{j\pi}} \; -1$$

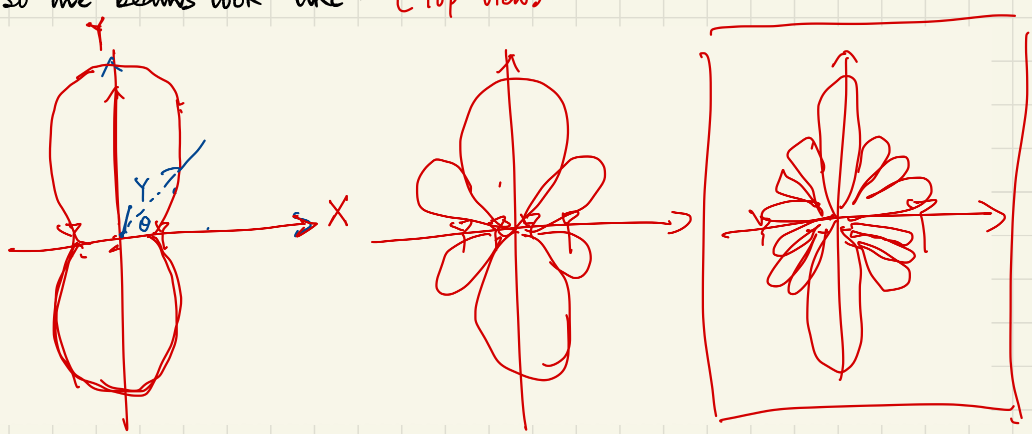
$$+)\, s_0 = s_0 e^{j0}$$

$$0$$

→ Plot Y_f or y_t against θ, here we plot when $d = \frac{\lambda}{2}$



→ So the beams look like: (Top view)



→ Observe, the natural beam is pointing towards Front and Back

antennas transmit same signal

Beam Rotation

Now I want the main lobe to point towards certain direction θ

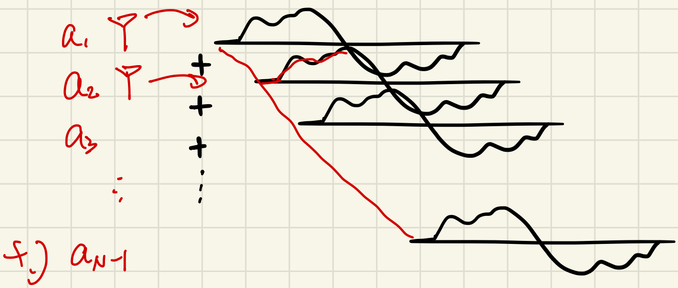
→ i.e., Max receive power towards θ.

→ How? By making signals from all antennas

So, first let's see how signals add up along θ

Recall $Y = \sum_{k=0}^{N-1} S_0 e^{jk\phi}$

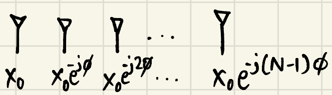
This is like



$= N S_0$

- For max SNR at R_x, compensate phase shift ϕ_i

i.e., $[e^{-j0} \; e^{-j\phi} \; e^{-j2\phi} \; \cdots \; e^{-j(N-1)\phi}] \; X_0$



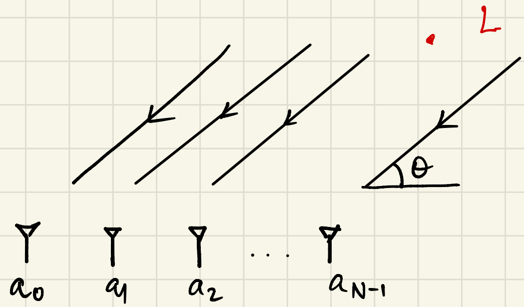
$$\therefore \; Y = \sum_{k=0}^{N-1} X_i e^{jk\phi} = (X_0 e^{-jk\phi}) e^{jk\phi}$$

$$\therefore \; Y = N X_0$$

This is called <u>Delay and Sum</u>

- Analogy: Stagger runners at the starting line with Δ cycle length different distance to ensure they all run the same distance

⊝ ANGLE OF ARRIVAL (AOA)



Signal arriving from θ (a certain direction)

Antenna array needs to figure out the Angle of Arrival (AoA) θ

Direction of Arrival (DoA)

How can you estimate AoA ? Well, similar concepts as beamforming

⊝ Say received signal is now

$$y_N = \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} \cos(2\pi f t) \\ \cos(2\pi f t + \phi) \\ \vdots \\ \cos(2\pi f t + (N-1)\phi) \end{bmatrix} \xrightarrow{\text{Freq.}} X(f) \begin{bmatrix} e^{j0} \\ e^{j\phi} \\ \vdots \\ e^{(N-1)\phi} \end{bmatrix}$$

⊝ From this received vector, how do you detect θ ?

↳ Answer: Try to delay and sum for all possible θ.

↳ Algorithm :

for $\theta_i = -\pi$ to π // search over all AoA θ

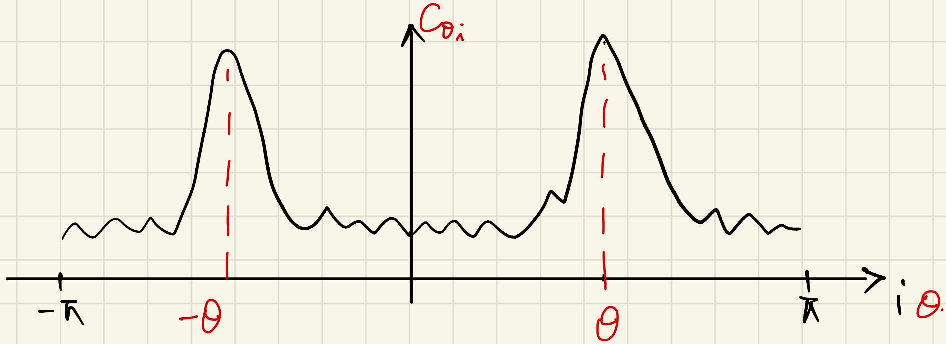
{

$\alpha_i = \frac{2\pi}{\lambda} d \cos\theta_i$ // calculate phase shift

$$C_{\theta_i} = \left[e^{-j0}\ e^{-j\alpha_i} \cdots e^{-j(N-1)\alpha_i} \right] \begin{bmatrix} Y_0 \\ \vdots \\ Y_{N-1} \end{bmatrix} = \sum e^{-jk\alpha_i} \cdot Y_i$$ // dot product

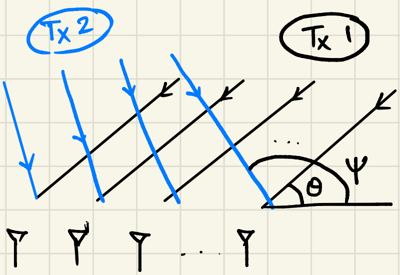
}

Plot (C_{θ_i}, θ_i) // Plot the AoA spectrum



$$\text{AoA} = \arg\max_{\theta} C_\theta$$

- Now, let's assume 2 transmitters sending in parallel.
  ↳ can we still decode the AoAs ?



say $\phi_1 = \frac{2\pi}{\lambda} d \cos\theta$

$\phi_2 = \frac{2\pi}{\lambda} d \cos\psi$

$$\underbrace{\begin{bmatrix} Y_0 \\ \\ \\ Y_{N-1} \end{bmatrix}}_{\text{Received signal}} = \underbrace{\begin{bmatrix} e^{j0} & e^{j0} \\ e^{j\phi_1} & e^{j\phi_2} \\ \vdots & \vdots \\ e^{j(N-1)\phi_1} & e^{j(N-1)\phi_2} \end{bmatrix}}_{\text{Steering Matrix}} \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{\text{source}}$$

$$\bar{Y} = A \, \bar{s}$$

- Now how can you decode θ and ψ
  ↳ Answer: Looking for a certain phase pattern $\longrightarrow \alpha_i$

$$\underbrace{\begin{bmatrix} e^{-j0} & e^{-j\alpha_i} & \cdots & e^{-j(N-1)\alpha_i} \end{bmatrix}}_{a_{\alpha_i}} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{N-1} \end{bmatrix} = \begin{bmatrix} e^{-j0} \cdot e^{j0} & e^{-j0} \cdot e^{j0} \\ e^{j(\phi_1-\alpha_i)} & e^{j(\phi_2-\alpha_i)} \\ \vdots & \vdots \\ e^{j(N-1)(\phi_1-\alpha_i)} & e^{j(N-1)(\phi_2-\alpha_i)} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Perform this for all values of α_i

Hope dot product large when $\alpha_i = \phi_1$ or $\alpha_i = \phi_2$

⊖ Modelling noise $\bar{Y} = A\bar{S} + n$

$$\begin{bmatrix} e^{j0} \\ \vdots \\ e^{j(N-1)\phi} \end{bmatrix}$$

$$\begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix} = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_d \\ | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} + \begin{bmatrix} n_0 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

Guess right

$$[-a_i-]^* \begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix} = [-a_i-]^* \begin{bmatrix} | & & \overset{30^\circ}{|} & & | \\ a_1 & \cdots & \boxed{a_i} & \cdots & a_d \\ | & & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix} + [-a_i-] \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

correlating for

Recall uncorrelated :

$$E[XY] = \underbrace{[x_1 \cdots x_n]^*}_{x} \overbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}}^{y}$$

$$= \sum_{k=1}^{N} x_k y_k$$

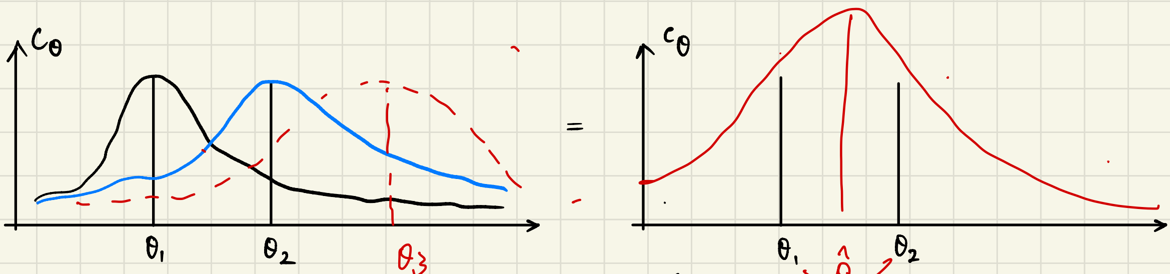
$$= x^* y = 0$$

$$= \underbrace{\|a_i\|^2 x_1}_{\substack{\text{Large} \\ N \text{ times } x_1}} + \underbrace{\sum_{j=1}^{d} a_i a_j x_j}_{\text{Hopefully small}} + \underbrace{a_i n}_{\text{uncorrelated}}$$

(Not always, especially when x_j is correlated w/ x_1

⊖ By correlating along all directions a_i, we get an

⊖ Problem is



$\times \quad \hat{\theta} \quad \times$

Peak is neither θ_1 nor θ_2

Especially when signals are correlated or $\theta_1 \simeq \theta_2$