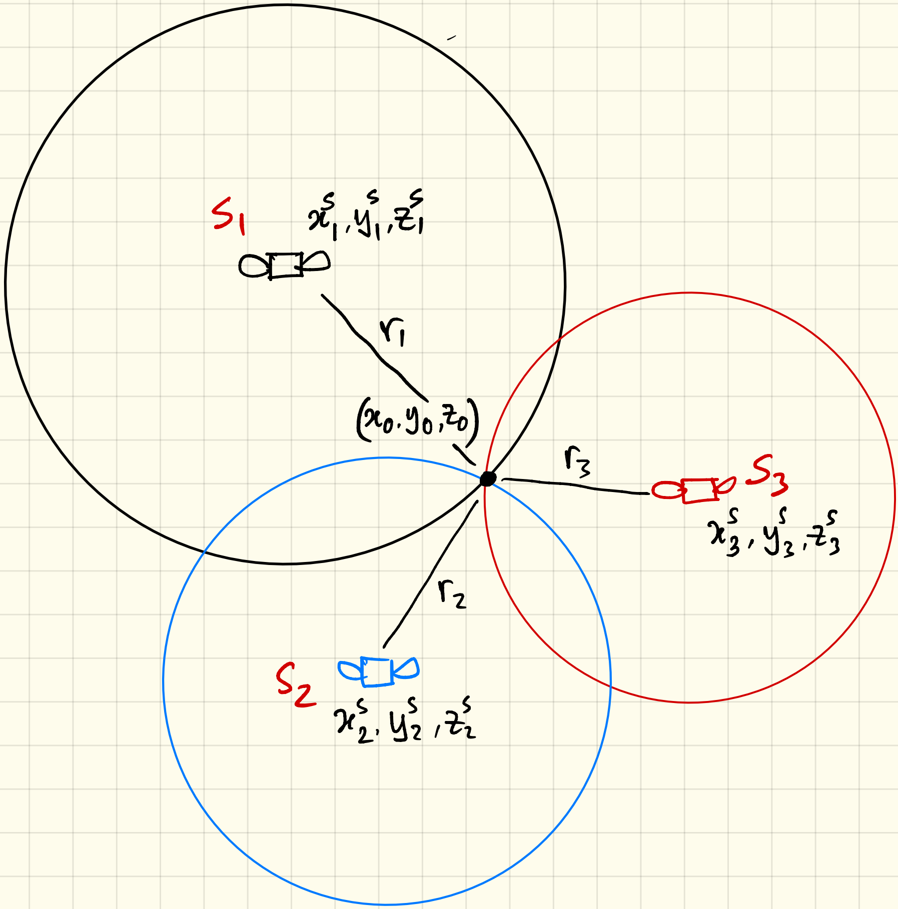
⑦ GPS localization : Basic idea.



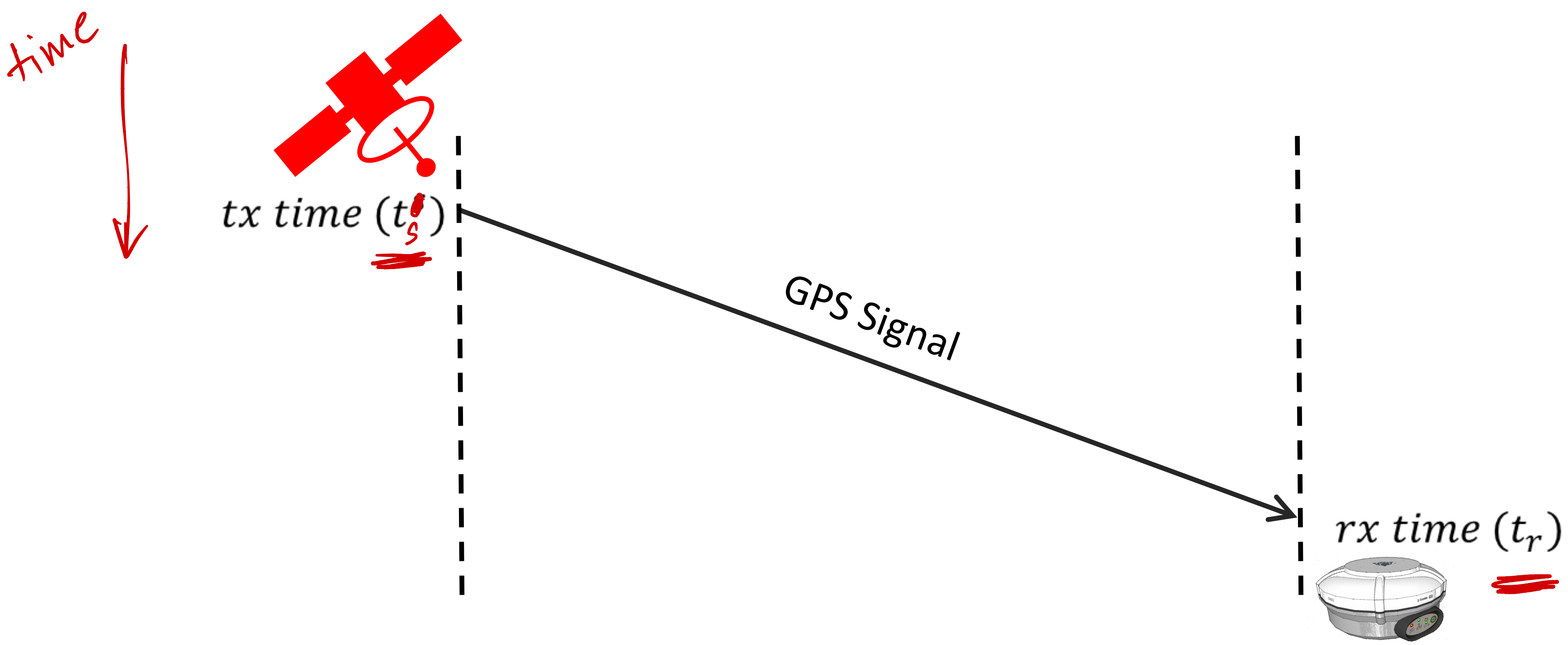
(a) Receiver measures dist. from itself to each satellite, r_i

(b) Formulates as $A\bar{x} = b$ problem, where $\bar{x} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$

(c) $\bar{b}$ does not fall in $C(A)$ ... thus needs to solve least squares

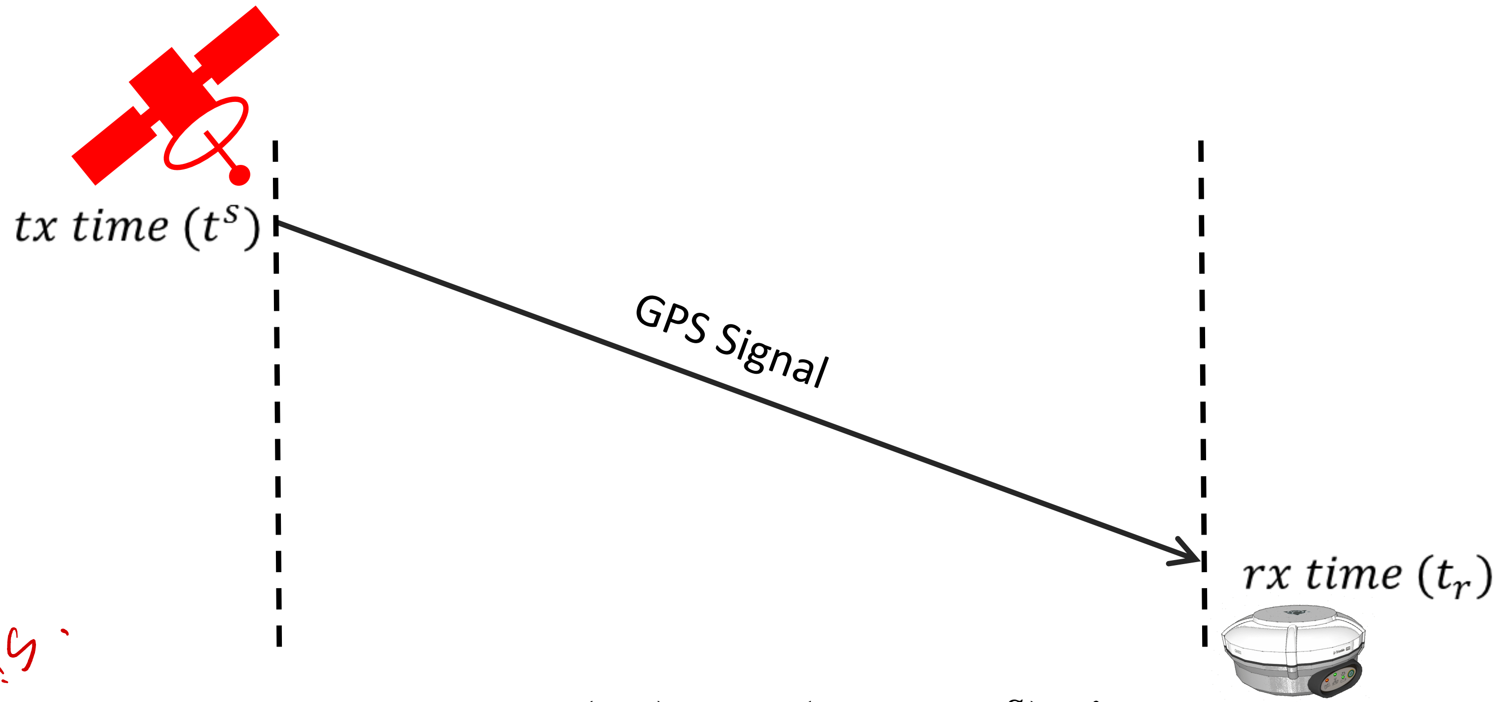
(d) GPS receiver's clock also gets clock synchronized to satellite

Basic GPS Localization



Distance $r_1 = (t_r - t_s) \times c$

Basic GPS Localization



$$Range(R) = (t_r - t^s) * c$$

non linear eq^ns.

$$\sqrt{(x_1^s - x_0)^2 + (y_1^s - y_0)^2 + (z_1^s - z_0)^2} = (t_r^{(1)} - t_s^{(1)}) \times c \quad (1)$$

$$\sqrt{(x_2^s - x_0)^2 + (y_2^s - y_0)^2 + (z_2^s - z_0)^2} = (t_r^{(2)} - t_s^{(2)}) \times c \quad (2)$$

$$\sqrt{x_3^s \quad '' \quad y_3^s \quad z_3^s} = (t_r^{(3)} - t_s^{(3)}) \times c \quad (3)$$

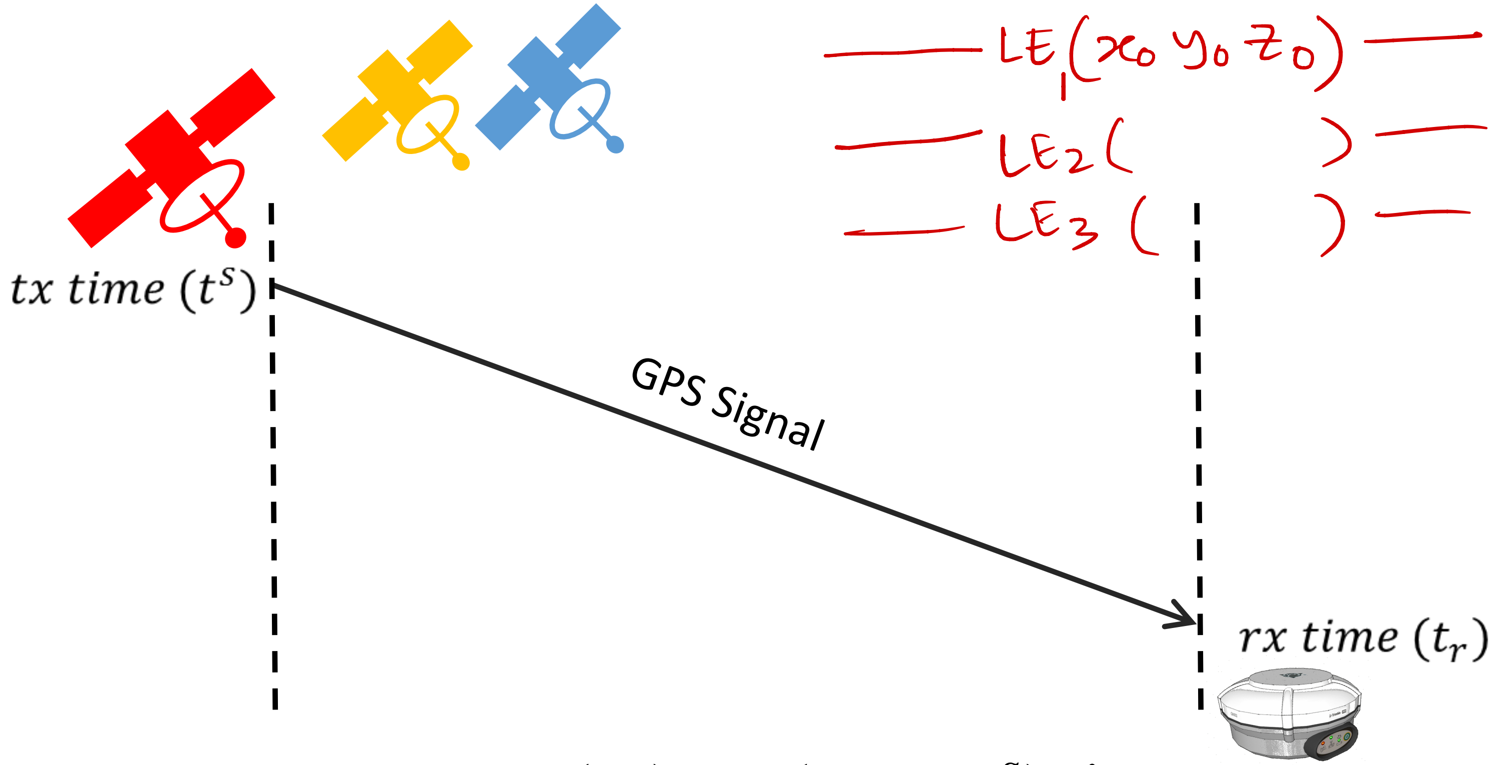
↳ Linearize the non linear eqⁿs.

Basic GPS Localization

$LE_1(x_0\ y_0\ z_0)$ — 1

$LE_2(\quad)$ — 2

$LE_3(\quad)$ — 3



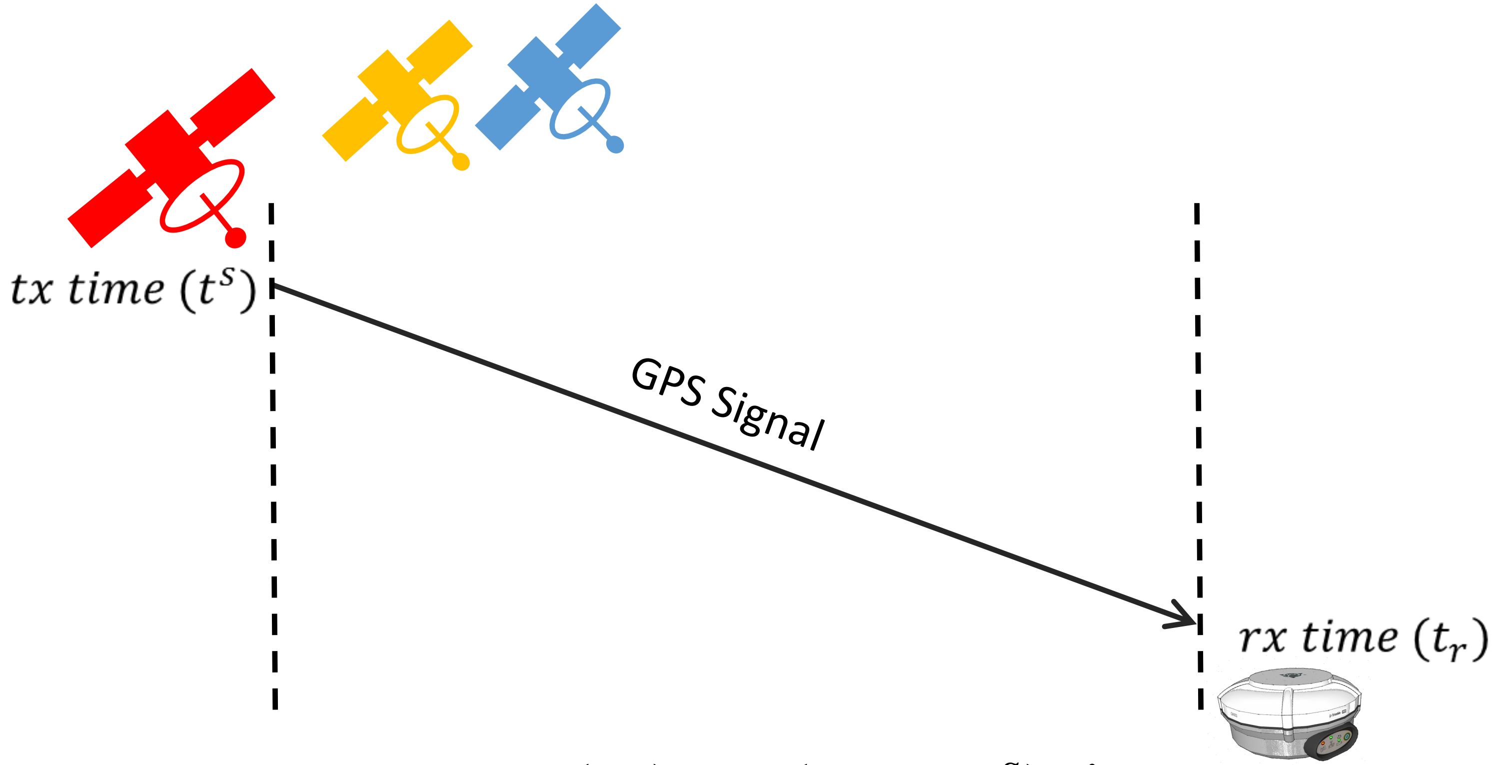
$$Range(R) = (t_r - t^s) * c$$

However, 3D location needs 3 equations … hence, use 3 satellites

Satellite geometry matrix

$$\underbrace{\begin{bmatrix} \text{Satellite locs.} \\ x_1^s \quad y_1^s \quad z_1^s \\ x_2^s \quad \cdots \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} (t_r^{(1)} - t_s^{(1)}) \times c \\ (t_r^{(2)} - t_s^{(2)}) \times c \\ (t_r^{(3)} - t_s^{(3)}) \times c \end{bmatrix}}_{b} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

Basic GPS Localization



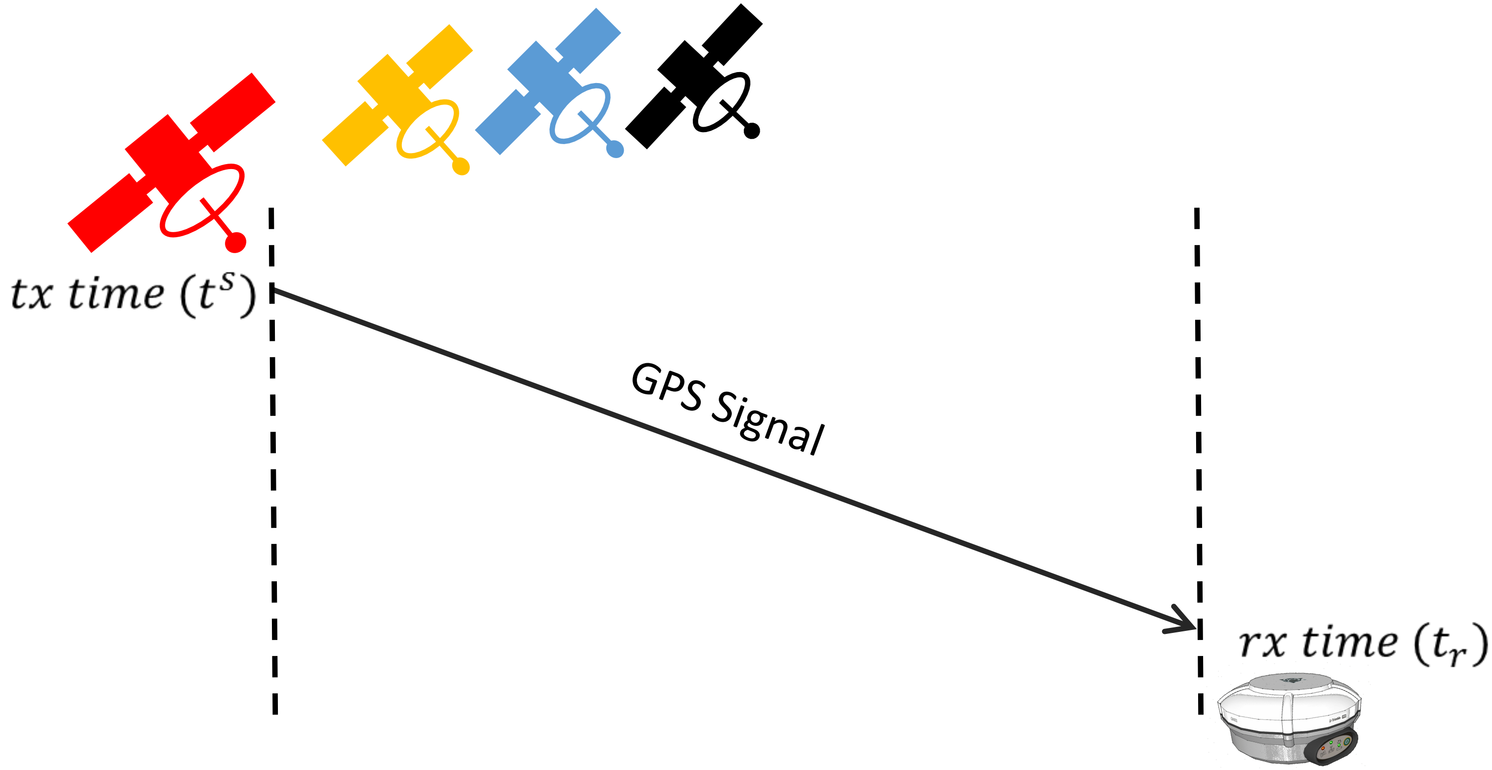
$$Range(R) = (t_r - t^s) * c$$

However, 3D location needs 3 equations … hence, use 3 satellites

$$\begin{bmatrix} \text{Satellite} \\ \text{Geometry} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

300 km

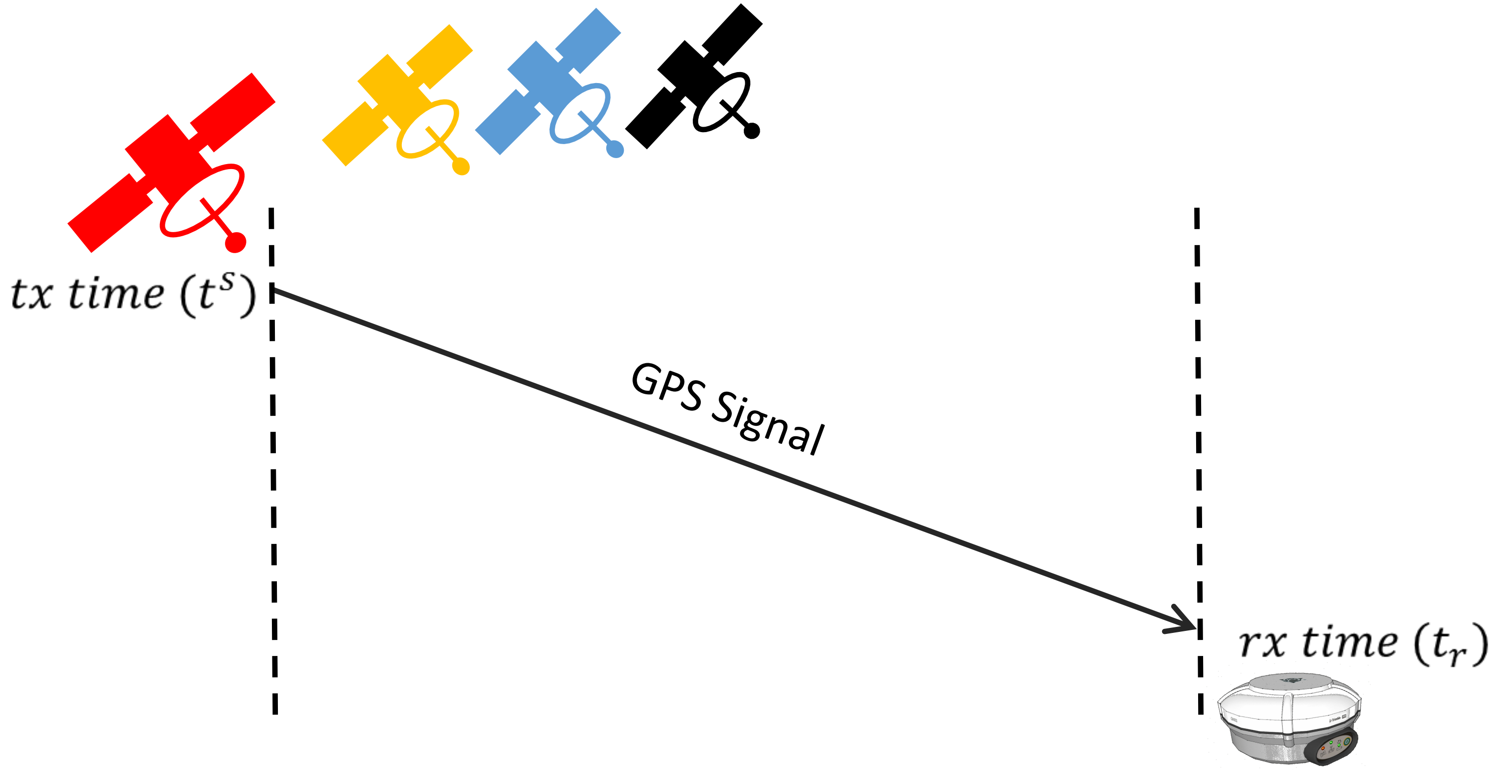
Basic GPS Localization



$$Range(R) = (t_r - t^s) * c + \delta_{clk} * c$$

New unknown δ … use 4th satellite and estimate both location and δ

Basic GPS Localization



$$Range(R) = (t_r - t^s) * c + \boxed{\delta_{clk} * c}$$

New unknown δ … use 4th satellite and estimate both location and δ

$$\begin{bmatrix} \text{Satellite} & c \\ \text{Geometry} & c \\ & c \\ \text{Matrix} & c \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \\ \delta_{clk} \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} \Rightarrow \boxed{\textbf{1-3m error}}$$

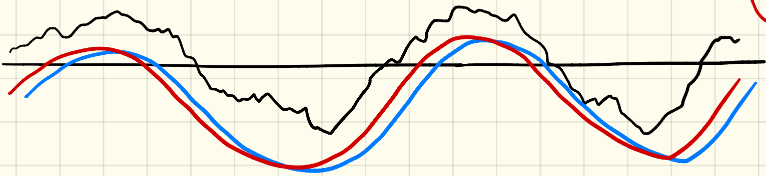
$$\underbrace{\sqrt{(x_1^s - x_0)^2 + (y_1^s - y_0)^2 + (z_1^s - z_0)^2} - \delta \cdot c}_{\text{Linearize .}} = \underbrace{(t_r - t_s) \cdot c}_{R_1}$$

$$\begin{bmatrix} \text{—— } LE(x_1^s \; y_1^s \; z_1^s) \text{ —— } -c \\ \text{——————} \\ \text{——————} \\ \text{——————} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ \delta \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}$$

→ Estimating receive time t_r at GPS hardware

↳ How to detect presence of a signal s in received signal Y.

Similar to detecting "Alexa" in sound signal



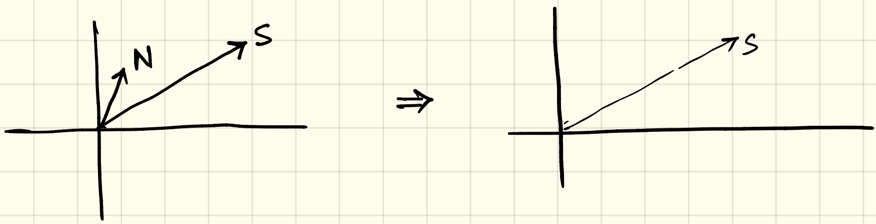
→ Given $y_5 \ y_6 \ y_7 \ y_8 \ y_9 \ \ldots$ Find $s_1 \ s_2 \ s_3 \ s_4 \ s_5$

→ Correlation :

$$C = \qquad = \frac{1}{K}$$

But $y_i = s_i$ because of

Then, $C = \frac{1}{K}[\text{—— } s \text{ ——}] \qquad = \frac{1}{K}$



→ If , Then decoding possible.

↳ Thus, S and N need to be

i.e.,

↳ In practice, how will K impact this E[S.N] ?

↳ better is decoding.

→ Not enough

What if the signal changes slowly ... then s will also match well with

Thus, should

with signal s[n]

└→ called property.

└→ Ideally,

→ Moreover :

↳ signal Z, expected from should exhibit to S.

└→ Otherwise Z will and GPS receiver will detect the

→ Summary: necessary for satellite signals :

1. Uncorrelated to noise
2. Good auto-correlation
3. Weak cross-correlation

GPS uses that satisfy these properties.