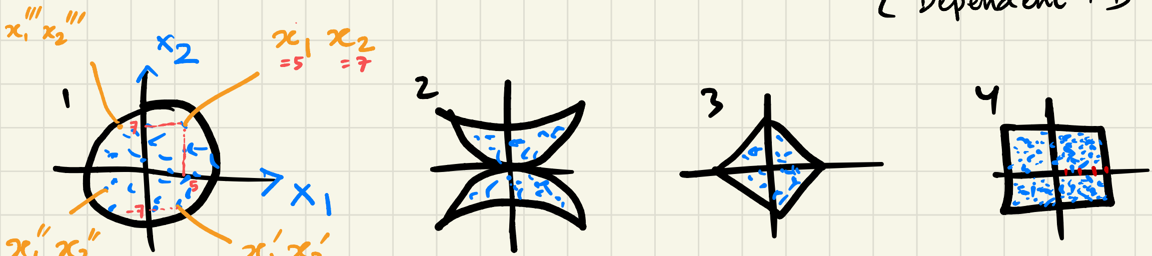
How about these data. Label them as
{ Correlated : C
{ Uncorrelated : U

{ Independent : I
{ Dependent : D

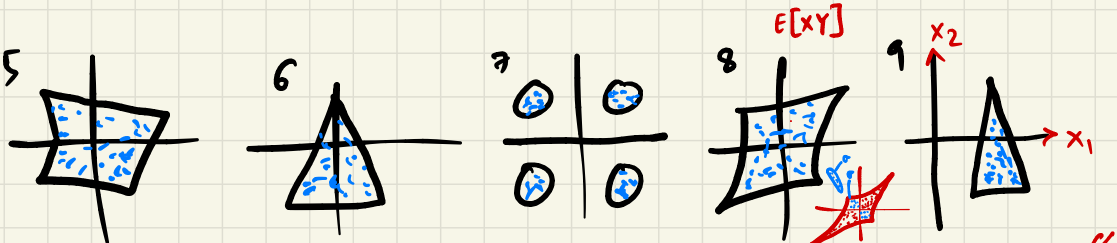


$E[X_1 X_2] = 0$ | Uncorrelated Dependent | Uncorrelated Dependent | Uncorr. Independent

$x_1 x_2 + x_1' x_2' + x_1'' x_2'' + \ldots\ldots = ?$

Yes, uncorrelated since they cancel out.

$P(x_2 | x_1)$ is $\neq$ $P(x_2)$ $\Rightarrow$ Not independent



Correlated Dependent | Uncorrelated Dep. | Uncorr Dep. | Corr. Dep. | Uncorr. Dep.

Indep. if $P(x_2|x_1) = P(x_2)$

$$\mathrm{Cov}(x_1, x_2) = \underbrace{E[x_1 x_2]}_{\text{Is this } = 0\,?} - \underbrace{E[x_1]}_{+ve}\underbrace{E[x_2]}_{-ve}$$

$E[x_1 x_2] \rightarrow E\left[(x_1 - \mu_{x_1})(x_2 - \mu_{x_2})\right]$

Application : covariance of received mic. data.

i.e., say $\boxed{x = A\bar{s} + \bar{n}}$ → DUET

$$\mathrm{Cov}(x) = E[xx^T]$$

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} | & | & & | \\ a_{\theta_1} & a_{\theta_2} & \cdots & a_{\theta_n} \\ | & | & & | \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_m \end{bmatrix}$$

$$\mathrm{Cov}(\bar{x}) = E[XX^T] = E\left[\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}[x_1\, x_2 \cdots x_m]\right]$$

$$= E\begin{bmatrix} x_1x_1 & x_1x_2 & \cdots & x_1x_m \\ x_2x_1 & x_2x_2 & \cdots & x_2x_m \\ \vdots & & & \vdots \end{bmatrix}$$

$$X_{m\times k} = \begin{bmatrix} x_1^{t_1} & x_1^{t_2} & \cdots & x_1^{t_k} \\ x_2^{t_1} & x_2^{t_2} & & x_2^{t_k} \\ \vdots & \vdots & & \vdots \\ x_m^{t_1} & x_m^{t_2} & & x_m^{t_k} \end{bmatrix}$$

$$\mathrm{cov}(X_{m\times k}) = E[\underbrace{XX^T}_{\text{matrix}}]$$

Because I zero mean the data.

$$\begin{bmatrix} x_1^{t_1} & x_1^{t_2} & \cdots & x_1^{t_K} \\ x_2^{t_1} & x_2^{t_2} & & x_2^{t_K} \\ \vdots & \vdots & & \vdots \\ x_m^{t_1} & x_m^{t_2} & & x_m^{t_K} \end{bmatrix} \begin{bmatrix} x_1^{t_1} & x_2^{t_1} & \cdots & x_m^{t_1} \\ x_1^{t_2} & x_2^{t_2} & \cdots & x_m^{t_2} \\ \vdots & & & \\ \vdots & & & \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & E[x_1 x_2] & & \\ & & & \\ & & & \end{bmatrix}$$