

Useful Formulae

A photon with a wavelength of 1 μm has an energy of 1.239 eV.

GaAs electron effective mass: $0.067m_0$

GaAs light hole effective mass: $0.12m_0$

GaAs heavy hole effective mass: $0.5m_0$

Free electron mass m_0 : 9.11×10^{-31} kg

Atomic weight of Ga: 69.723 g/mole; Atomic weight of As: 74.922 g/mole

Atomic weight of Al: 26.982 g/mole; Atomic weight of In: 114.818 g/mole

Atomic weight of P: 30.974 g/mole

Avogadro's Number: 6.022×10^{23} molecules/mole

Boltzmann's Constant: 1.38×10^{-23} J/K = 8.62×10^{-5} eV/K

Plank's Constant: 6.626×10^{-34} J-s = 4.136×10^{-15} eV-s

Free Space Permittivity: $\epsilon_0 = 8.85 \times 10^{-14}$ F/cm = 8.85×10^{-12} F/m (use 9×10^{-12} F/m for hand calculation)

1 eV = 1.602×10^{-19} Joules; $q = 1.6 \times 10^{-19}$ C; Speed of Light $c = 2.998 \times 10^{10}$ cm/s

Bohr Radius: 0.5292 Å; Rydberg: 13.6 eV; $1/e \sim 0.37$

$kT = 0.0259$ eV @ Room Temperature; $kT = 0.025$ eV @ 290K; $\ln(x) = 2.3 \log(x)$; $\ln(10) = 2.3$

	III A	IV A	V A	VIA
	5 B	6 C	7 N	8 O
	13 Al	14 Si	15 P	16 S
II B	30 Zn	31 Ga	32 Ge	33 As
	48 Cd	49 In	50 Sn	51 Sb
			52 Te	

$$p = m^* v = \hbar k = \frac{h}{\lambda} \quad E = hv = \hbar \omega \quad E = \frac{1}{2} m^* v^2 = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \cong e^{-(E-E_F)/kT} \text{ for } E \gg E_F \quad N(E) = DOS(E) = \frac{\sqrt{2}}{\pi^2} \left(\frac{m^*}{\hbar^2} \right)^{3/2} E^{1/2}$$

$$n_0 = \int_{E_c}^{\infty} f(E) N(E) dE \quad \text{Triangle Area: } A = 0.5 \times (\text{base}) \times (\text{height}) \quad m^* = \frac{\hbar^2}{a^2 E / dk^2}$$

Note: $N(E)$ is referenced to the bottom of the conduction band for electrons and the top of the valence band for holes.

$$p_0 = N_V [1 - f(E_V)] = N_V e^{-(E_F - E_V)/kT} \quad \frac{D}{\mu} = \frac{kT}{q} \quad \mu = \frac{q\hbar}{m^*} \quad n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$$

$$N_C = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{3/2}; \quad N_V = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{3/2} \quad n_i = N_C e^{-(E_C - E_i)/kT} \quad p_i = N_V e^{-(E_i - E_V)/kT}$$

$$\text{For } ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad R = \frac{\rho L}{wt} = \frac{L}{\sigma wt}$$

$$n_0 = n_i e^{(E_F - E_i)/kT} \quad p_0 = n_i e^{(E_i - E_F)/kT} \quad n_0 p_0 = n_i^2 \quad \sigma = q(n\mu_n + p\mu_p)$$

$$n = N_C e^{-(E_C - E_F)/kT} = n_i e^{(E_F - E_i)/kT} \quad p = N_V e^{-(E_F - E_V)/kT} = n_i e^{(E_i - E_F)/kT} \quad np = n_i^2 e^{(E_F - E_F)/kT}$$

$$\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx} = \frac{1}{q} \frac{dE_i}{dx} \quad \frac{d\mathcal{E}(x)}{dx} = -\frac{d^2\mathcal{V}(x)}{dx^2} = \frac{\rho(x)}{\epsilon} = \frac{q}{\epsilon} (p - n + N_d^+ - N_a^-)$$

$$\delta n = g_0 \tau_n; \quad \delta p = g_0 \tau_p \quad \delta p(t) = \delta p(0) e^{-\alpha_r(p_0 + n_0)t} = \delta p(0) e^{-t/\tau_p} \quad J_{drift} = \sigma \mathcal{E}$$

$$p - n + N_d^+ - N_a^- = 0 \quad \leftarrow (\text{neutral or quasi-neutral regions})$$

$$J_n(x) = q\mu_n n(x)\mathcal{E}(x) + qD_n \frac{dn(x)}{dx} \quad J_p(x) = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$$

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p} \quad \frac{\partial n(x,t)}{\partial t} = \frac{\partial \delta n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

$$L = \sqrt{D\tau} \quad J_{total} = J_n + J_p$$

$$\text{Continuity:} \quad \frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p} \quad \frac{\partial \delta n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

$$\text{Steady State Diffusion:} \quad \frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} = \frac{\delta n}{L_n^2} \quad \frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} = \frac{\delta p}{L_p^2}$$

$$V_0 = \frac{kT}{q} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \frac{N_a}{(n_i^2 / N_d)} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} \quad \frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$$

$$qV_0 = E_{vp} - E_{vn} \quad -V_0 = \int_{-x_{p0}}^{x_{n0}} \mathcal{E}(x) dx$$

$$\text{Short Base Diode:} \quad I_p(x_n) = AJ_p(\text{diff}) = -AqD_p \frac{dp(x_n)}{dx_n} \approx qAD_p \frac{\Delta p_n}{\ell} \quad Q_p = \frac{1}{2} qA\ell \Delta p_n$$

$$W = \left[\frac{2\mathcal{E}(V_0 - V)}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} \quad qAx_{p0}N_a = qAx_{n0}N_d$$

$$\Delta p_n = p(x_{n0}) - p_n = p_n (e^{qV/kT} - 1) \quad \Delta n_p = n(-x_{p0}) - n_p = n_p (e^{qV/kT} - 1)$$

$$\delta p(x) = \Delta p_n e^{-x/L_p} \quad \delta n(x) = \Delta n_p e^{-x/L_n} \quad (\text{referenced to depletion region edge})$$

$$I = qA \left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right) (e^{qV/kT} - 1) = I_0 (e^{qV/kT} - 1) \quad C = \left| \frac{dQ}{dV} \right|$$

$$I_{op} = qAg_{op}(L_p + L_n + W) \quad I = I_0 (e^{qV/kT} - 1) - I_{op} \quad f \cdot f \cdot = \frac{I_m V_m}{I_{sc} V_{oc}}$$

$$C_j = \epsilon A \left[\frac{q}{2\mathcal{E}(V_0 - V)} \frac{N_a N_d}{N_a + N_d} \right]^{1/2} = \frac{\epsilon A}{W}$$

$$Q_p = qA \int_0^\infty \delta p(x_n) dx_n = qA \Delta p_n \int_0^\infty e^{-x_n/L_p} dx_n = qAL_p \Delta p_n$$

$$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA \frac{L_p}{\tau_p} \Delta p_n = qA \frac{D_p}{L_p} p_n (e^{qV/kT} - 1) \quad G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p} \frac{d}{dV} (e^{qV/kT}) = \frac{q}{kT} I$$

n-Channel MOS:

$$C_i = \frac{\epsilon_i}{d} \quad C_d = \frac{\epsilon_s}{W} \quad C = \frac{C_i C_d}{C_i + C_d} \quad V_T = \Phi_{ms} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F$$

$$\phi_s(inv.) = 2\phi_F = 2 \frac{kT}{q} \ln \frac{N_a}{n_i} \quad W = \left[\frac{2\epsilon_s \phi}{qN_a} \right]^{1/2} \quad Q_d = -qN_a W_m = -2(\epsilon_s q N_a \phi_F)^{1/2}$$

At V_{FB} : $C_{FB} = \frac{C_i C_{debye}}{C_i + C_{debye}} \quad L_D = \sqrt{\frac{\epsilon_s kT}{q^2 p_0}} \quad C_{debye} = \frac{\epsilon_s}{L_D}$

$$I_D = \frac{\bar{\mu}_n Z C_i}{L} \left\{ \left(V_G - V_{FB} - 2\phi_F - \frac{1}{2} V_D \right) V_D - \frac{2}{3} \frac{\sqrt{2\epsilon_s q N_a}}{C_i} \left[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2} \right] \right\}$$

$$I_D \approx \frac{\bar{\mu}_n Z C_i}{L} \left\{ (V_G - V_T) V_D - \frac{1}{2} V_D^2 \right\} \quad g_m = \frac{\partial I_D}{\partial V_G}$$

$$g_m(sat.) = \frac{\partial I_D(sat.)}{\partial V_G} \approx \frac{Z}{L} \bar{\mu}_n C_i (V_G - V_T)$$

Saturation: $I_D(sat.) \approx \frac{1}{2} \bar{\mu}_n C_i \frac{Z}{L} (V_G - V_T)^2 = \frac{Z}{2L} \bar{\mu}_n C_i V_D^2(sat.)$

$$V_T = \Phi_{mn} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F = V_{FB} - \frac{Q_d}{C_i} + 2\phi_F$$

$$V_T = \left| \begin{array}{c|c|c|c} \Phi_{ms} & -\frac{Q_i}{C_i} & -\frac{Q_d}{C_i} & +2\phi_F \\ \hline (-) & (-) & (+) \text{ n channel} \\ & & (-) \text{ p channel} & (+) \text{ n channel} \\ & & & (-) \text{ p channel} \end{array} \right|$$

Note: The first two terms on the right-hand side of this expression correspond to the flatband voltage.

p-Channel MOS: For a p-channel device ϕ_F is negative and Q_d is positive.

p-n-p BJT:

$$I_{E_p} = qA \frac{D_p}{L_p} \left(\Delta p_E \operatorname{ctnh} \frac{W_b}{L_p} - \Delta p_C \operatorname{csc} h \frac{W_b}{L_p} \right) \quad I_C = qA \frac{D_p}{L_p} \left(\Delta p_E \operatorname{csc} h \frac{W_b}{L_p} - \Delta p_C \operatorname{ctnh} \frac{W_b}{L_p} \right)$$

$$I_B = qA \frac{D_p}{L_p} \left[\left(\Delta p_E + \Delta p_C \right) \tanh \frac{W_b}{2L_p} \right]$$

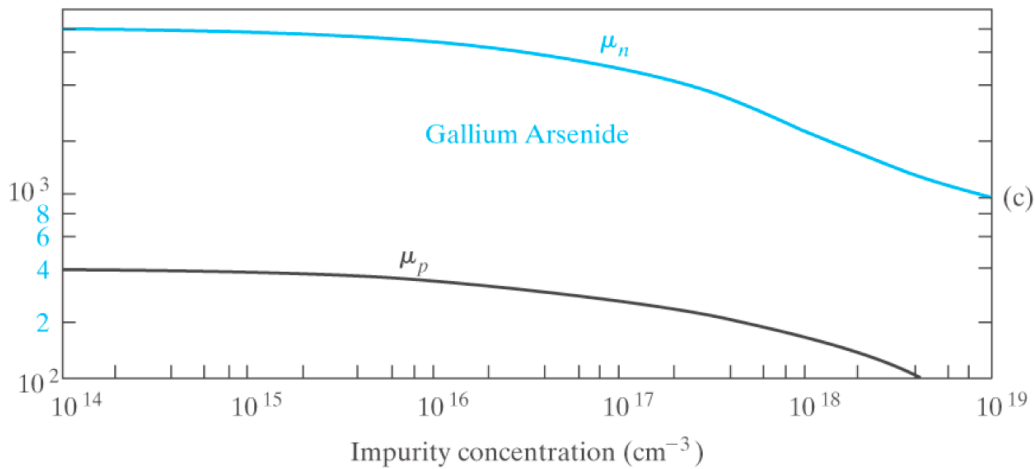
$$\Delta p_E = p_n \left(e^{qV/kT} - 1 \right) \quad \Delta p_C = p_n \left(e^{qV_{CB}/kT} - 1 \right) \quad I_B(\operatorname{recomb}) = \frac{Q}{\tau}$$

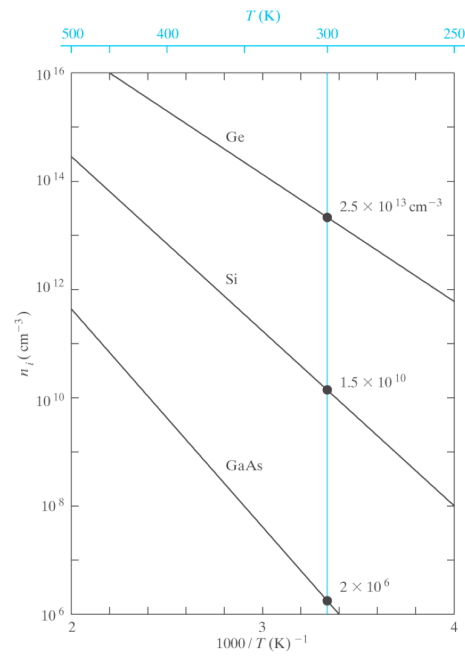
$$B = \frac{I_C}{I_{E_p}} = \frac{\operatorname{csc} h \frac{W_b}{L_p}}{\operatorname{ctnh} \frac{W_b}{L_p}} = \sec h \frac{W_b}{L_p} \approx 1 - \left(\frac{W_b^2}{2L_p^2} \right) \quad \frac{i_C}{i_E} = B\gamma \equiv \alpha$$

$$\gamma = \frac{I_{E_p}}{I_{E_n} + I_{E_p}} = \left[1 + \frac{L_p^n n_n \mu_p}{L_n^p p_p \mu_n} \tanh \frac{W_b}{L_p} \right]^{-1} \approx \left[1 + \frac{W_b n_n \mu_p}{L_n^p p_p \mu_n} \right]^{-1}$$

$$\frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1 - \alpha} \equiv \beta \quad \frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t} \quad (\text{for } \gamma = 1)$$

$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{EB}} \approx \frac{q}{kT} I_B \quad i_C = \beta i_B = -\beta \frac{v_{in}}{r_\pi} \quad v_{out} = i_C R_C = -\beta \frac{R_C}{r_\pi} v_{in}$$





		E_g (eV)	μ_n (cm ² /V-s)	μ_p (cm ² /V-s)	m_n^*/m_0 (m_l, m_t)	m_p^*/m_0 (m_l, m_t)	a (Å)	ϵ_r	Density (g/cm ³)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11.0	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10 ⁵	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

All values at 300 K.

*Vaporizes

