ECE 329 Tutorial 17

(2019/11/18)

Reflection Coefficient at Load:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection Coefficient at Source:

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

Voltage Standing Wave Ratio (VSWR):

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Input Impedance (seen by the source):

$$Z_{in} = Z_0 \frac{Z_L + Z_0 tanh(\gamma L)}{Z_0 + Z_L tanh(\gamma L)}$$

Where γ is the complex wave number:

$$\gamma = \alpha + j\beta$$

If we assume lossless Transmission Line, then $\alpha = 0$ and $\gamma = j\beta$, and the input impedance for the case of lossless Transmission Line becomes:

$$Z_{in(Lossless)} = Z_0 \frac{Z_L + jZ_0 tan(\beta L)}{Z_0 + jZ_L tan(\beta L)}$$

Where L is the length of Transmission Line, and β the propagation wave number along the line.

Problem 1 (Warm-Up Calculation):

A source with source impedance $Z_S = 50\Omega$ drives a transmission line with characteristic impedance $Z_0 = 50\Omega$ that is $\frac{1}{8}$ of a wavelength long, and is terminated by a load with load impedance $Z_L = 50 - 25j\ \Omega$.

(a) Calculate the reflection coefficient at load Γ_L.

(b) Calculate the Voltage Standing Wave Ratio (VSWR).

(c) Calculate the input impedance seen by the source, assuming lossless transmission line.

Problem 2 (Calculation with Input Impedance):

The input impedance for a 30-cm length of lossless transmission line with characteristic impedance $Z_0 = 100\Omega$ operating at frequency $f = 2GHz$ is $Z_{in} = 92.3 - 67.5j\ \Omega$. The propagation phase velocity is 0.7 c. Determine the load impedance.

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Problem 1:

(a) $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-25j}{100-25j} = \frac{25e^{-j\frac{\pi}{2}}}{\sqrt{100^2+25^2}\, e^{-j\arctan(\frac{1}{4})}} = \frac{25}{\sqrt{100^2+25^2}} e^{j(\arctan(\frac{1}{4})-\frac{\pi}{2})}$

(b) $|\Gamma_L| = \frac{25}{\sqrt{100^2+25^2}} \Rightarrow VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$

(c) $\beta L = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4} \Rightarrow Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta L)}{Z_0 + jZ_L\tan(\beta L)}$

$= Z_0 \frac{Z_L + jZ_0}{Z_0 + jZ_L}$, with $Z_L = 50 - 25j\ \Omega$

$Z_0 = 50\ \Omega$

Problem 2:

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p} = \frac{2\pi \cdot 2\times10^9}{0.7\cdot 3\times10^8} = \frac{40\pi}{2.1} \Rightarrow \beta L = \frac{40\pi}{2.1}\times 0.3 = \frac{40\pi}{7}$

We have: $Z_{in} = 92.3 - 67.5j = Z_0 \frac{Z_L + jZ_0\tan(\beta L)}{Z_0 + jZ_L\tan(\beta L)}$

$$0.923 - 0.675j = \frac{Z_L + 100j\tan(\frac{40\pi}{7})}{100 + Z_L j\tan(\frac{40\pi}{7})}$$

Solve for Z_L, we have: $Z_L = 49.997 + 0.015j\ \Omega$

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Reflection Coefficient at Load:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Input Impedance (seen by the source):

$$Z_{in} = Z_0 \frac{Z_L + Z_0 tanh(\gamma L)}{Z_0 + Z_L tanh(\gamma L)}$$

Where γ is the complex wave number:

$$\gamma = \alpha + j\beta$$

If we assume lossless Transmission Line, then $\alpha = 0$ and $\gamma = j\beta$, and the input impedance for the case of lossless Transmission Line becomes:

$$Z_{in(Lossless)} = Z_0 \frac{Z_L + jZ_0 tan(\beta L)}{Z_0 + jZ_L tan(\beta L)}$$

Where L is the length of Transmission Line, and β the propagation wave number along the line.

Input Voltage:

$$\tilde{V}_{in} = \tilde{V}_s \frac{Z_{in}}{Z_s + Z_{in}}$$

The voltage phasor at any point on the Transmission Line (starting at $z = -L$, terminating at $z = 0$ by the load) is:

$$\tilde{V}(z) = \tilde{V}_0^+ (e^{-\gamma z} + \Gamma_L e^{+\gamma z})$$

It is easy to see that:

$$\tilde{V}_{in} = \tilde{V}(z = -L) \qquad \tilde{V}_{Load} = \tilde{V}(z = 0)$$

Problem 1 (Warm-Up):

The reflection coefficient at the load for a Transmission Line with $Z_0 = 50\Omega$ is measured as $\Gamma_L = 0.516e^{j8.2^\circ}$ at a certain operation frequency f. Find the load impedance Z_L.

Problem 2 (Find the Voltage Across the Load):

Consider the lossless Transmission Line shown in the figure below. The source voltage is $V_S(t) = 10cos\left(\omega t + \frac{\pi}{6}\right) V$ with source impedance $Z_s = 25\Omega$. The Transmission Line has a characteristic impedance of $Z_0 = 50\Omega$ and length of $L = \frac{\lambda}{4}$, terminated by the load with load impedance of $Z_L = 100\Omega$. Find the voltage across the load as a function of time ($V_L(t)$).

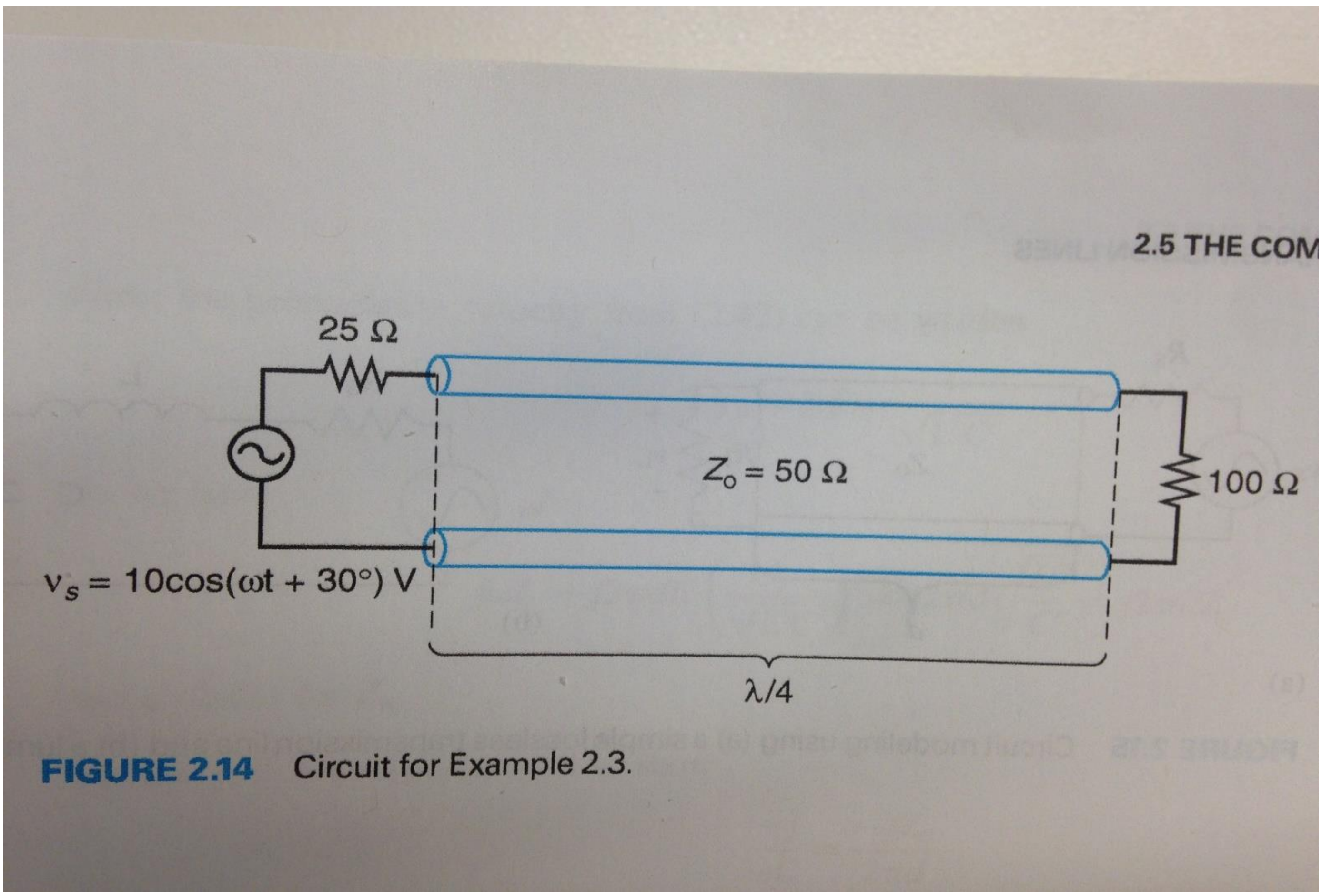

FIGURE 2.14 Circuit for Example 2.3.

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Problem 1:

$$V_S(t) = 10\cos(\omega t + \frac{\pi}{6}) \Rightarrow \tilde{V}_s = 10 e^{j\frac{\pi}{6}}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50}{150} = \frac{1}{3}$$

$$\beta L = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow \tan(\beta L) = \tan(\frac{\pi}{2}) = \infty$$

we have: $Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta L)}{Z_0 + jZ_L\tan(\beta L)}$ (assuming lossless $\gamma \to j\beta$)

$$= Z_0 \frac{Z_0}{Z_L}$$

$$= 25\,\Omega$$

consequently, we have: $\tilde{V}_{in} = \tilde{V}_s \frac{Z_{in}}{Z_{in} + Z_s} = \frac{1}{2}\tilde{V}_s = 5 e^{j\frac{\pi}{6}}$

$$\tilde{V}_{in} = \tilde{V}(z=-L) = \tilde{V}_0^+ (e^{-\gamma z} + \Gamma_L e^{\gamma z})|_{z=-L}$$

$$= \tilde{V}_0^+ (e^{\gamma L} + \Gamma_L e^{-\gamma L})$$

$$= \tilde{V}_0^+ (e^{j\beta L} + \Gamma_L e^{-j\beta L}), \text{ since lossless} \to \gamma = j\beta$$

$$= \tilde{V}_0^+ (e^{j\frac{\pi}{2}} + \Gamma_L e^{-j\frac{\pi}{2}})$$

$$= \tilde{V}_0^+ (\frac{2}{3}j)$$

$$\tilde{V}_0^+ = \frac{3\tilde{V}_{in}}{2j} = \frac{3\tilde{V}_{in}}{2e^{j\frac{\pi}{2}}} = \frac{15}{2} e^{-j\frac{\pi}{3}}$$

Therefore, $\tilde{V}_L = \tilde{V}(z=0) = \tilde{V}_0^+ (e^{-\gamma z} + \Gamma e^{\gamma z})\big|_{z=0}$

$$= \tilde{V}_0^+ (1 + \Gamma_L)$$

$$= \frac{4}{3} \tilde{V}_0^+$$

$$= 10\, e^{-j\frac{\pi}{3}} \Rightarrow V_L(t) = Re\{\tilde{V}_L e^{j\omega t}\}$$

$$= 10 \cos(\omega t - \frac{\pi}{3})\ V$$

Problem 2:

$\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_L = \dfrac{Z_0(1+\Gamma_L)}{1-\Gamma_L}$, with $Z_0 = 50\,\Omega$ and $\Gamma_L = 0.516\, e^{j8.2°}$

⇓

$$Z_L = 108.93 + 66.58j\ \Omega$$