HW Solutions

ECE 310: Digital Signal Processing, Spring 2026

Due Date: 04/03/26
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Solution:

We can see that $y[n]$ is a circularly shifted version of $x[n]$.

$y[n]$ is $x[n]$ circularly shifted to the left by 2 samples. This can be written as $y[n] = x((n+2))_6 = x((n-(-2)))_6$.

Use the circular shift property of the DFT: $x((n-n_0))_N \xrightarrow{DFT} e^{-j\frac{2\pi k n_0}{N}} X[k]$.

Here, $n_0 = -2$ and $N = 6$, and $Y[k]$ is the DFT of $y[n]$, therefore:

$$Y[k] = e^{-j\frac{2\pi k(-2)}{6}} X[k] = e^{j\frac{4\pi k}{6}} X[k] = e^{j\frac{2\pi k}{3}} X[k]$$

So, in terms of $X_0, \ldots, X_5$:

$$Y[k] = e^{j\frac{2\pi k}{3}} X_k \quad \text{for } k = 0, 1, \ldots, 5$$

Grading: *6 points*

- -0: Correct answer
- -2: Minor mistake: Final step ($n_o = -2$ and $N = 6$) is off
- -4: Major mistake: Failed to identify circuilar shift DFT property, effort shown
- -6: Empty or invalid

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Solution:

(a)

By inspection we see

$$Y[k] = X[< k - 4 >_8]$$

Thus,

$$\begin{aligned} y[n] &= e^{j\frac{2\pi \cdot 4}{8} n} x[n] \\ &= e^{j\pi n} x[n] \\ &= \{x_0, -x_1, x_2, -x_3, x_4, -x_5, x_6, -x_7\} \end{aligned}$$

(b)

We can see

$$Z[k] = X[k] + Y[k]$$

Thus,

$$\begin{aligned} z[n] &= x[n] + y[n] \\ &= \{x_0 + x_0, x_1 - x_1, x_2 + x_2, x_3 - x_3, x_4 + x_4, x_5 - x_5, x_6 + x_6, x_7 - x_7\} \\ &= \{2x_0, 0, 2x_2, 0, 2x_4, 0, 2x_6, 0\} \end{aligned}$$

Grading: *16 Points*

- +8: Parts (a), (b)
  - -0: correct
  - -2: Minor mistake: minor numerical mistake in final answer, correct property adopted
  - -5: Major mistake: failed to identify the property to use, the result is off, effort shown
  - -8: Empty or invalid

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Solution:

Signal $x[n]$ has length $N_x = 18$. We zero-pad it to get $y[n]$ of length $N_y = 18 + 12 = 30$.

Key Relationships:

- Zero-padding does not change the DTFT, so $X_d(\omega) = Y_d(\omega)$ for all ω.
- The DFT is a sampled version of the DTFT.

$$X[k] = X_d\left(\frac{2\pi k}{18}\right) = X_d\left(\frac{\pi k}{9}\right)$$

$$Y[k] = Y_d\left(\frac{2\pi k}{30}\right) = Y_d\left(\frac{\pi k}{15}\right)$$

(a) True

$X[0] = X_d(0)$ and $Y[0] = Y_d(0)$. Since $X_d(\omega) = Y_d(\omega)$, this is true.

(b) False

$X[1] = X_d(\pi/9)$ and $Y[1] = Y_d(\pi/15)$.

(c) True

$X[3] = X_d\left(\frac{\pi \cdot 3}{9}\right) = X_d\left(\frac{\pi}{3}\right)$. $Y[5] = Y_d\left(\frac{\pi \cdot 5}{15}\right) = Y_d\left(\frac{\pi}{3}\right)$.
Since $X_d(\omega) = Y_d(\omega)$, this is true since they are evaluated at $\omega = \frac{\pi}{3}$.

(d) True

Zero-padding does not change the DTFT. $X_d(\omega) = Y_d(\omega)$.

(e) False

Zero-padding does not change the DTFT. If ω for X and Y are different, then they are not equal.

(f) True

By the definition of the relationship between the DFT and the DTFT. $Y[k] = Y_d\left(\frac{2\pi k}{N_y}\right) = Y_d\left(\frac{2\pi k}{30}\right) = Y_d\left(\frac{\pi k}{15}\right)$.

Grading: *36 points*

- +6: Parts (a) - (f)
  - -0: Correct
  - -3: Correct T/F without justification
  - -6: Incorrect TF, empty, or invalid

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Solution:

(a)

We are given that there are 3 cosines that comprise this signal and the DFT plot shows 3 distinct pairs of peaks, therefore there is no spectral leakage. Thus, the digital frequencies corresponding to the indices of each peak are the frequencies of the cosines.

The indices of a DFT correspond to digital frequencies via

$$\omega = \frac{2\pi}{N}k, \quad 0 \le k < N$$

where N is the length of the signal.

Therefore,

$$\omega_1 = \frac{2\pi}{16} \cdot 1 = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi}{16} \cdot 2 = \frac{\pi}{4}, \quad \omega_3 = \frac{2\pi}{16} \cdot 4 = \frac{\pi}{2}$$

(b)

The magnitude for the DFT of a finite-length cosine is given by

$$|X_0[k]| = |A_0 \frac{\sin\left(\frac{N}{2}\left(\frac{2\pi k}{N} - \omega_0\right)\right)}{2\sin\left(\frac{1}{2}\left(\frac{2\pi k}{N} - \omega_0\right)\right)} e^{-j\frac{N-1}{2}\left(\frac{2\pi k}{N} - \omega_0\right)} + A_0 \frac{\sin\left(\frac{N}{2}\left(\frac{2\pi k}{N} + \omega_0\right)\right)}{2\sin\left(\frac{1}{2}\left(\frac{2\pi k}{N} + \omega_0\right)\right)} e^{-j\frac{N-1}{2}\left(\frac{2\pi k}{N} + \omega_0\right)}|$$

Which reduces to $|X[k]| = \frac{AN}{2}$ when evaluated at the index corresponding to the peak of the main lobe, whose location is the frequency of the cosine.

Thus we can evaluate the cosine amplitudes as

$$A_1 = 8 \cdot \frac{2}{16} = 1, \quad A_2 = 4 \cdot \frac{2}{16} = \frac{1}{2}, \quad A_3 = 6 \cdot \frac{2}{16} = \frac{3}{4}$$

Grading: *18 points*

- +9: (a)
  - -0: Correct
  - -3: One frequency (ω) is wrong
  - -6: Two frequencies (ω) are wrong
  - -9: Three frequencies (ω) are wrong
- +9: (b)
  - -0: Correct
  - -3: One amplitude (A) is wrong
  - -6: Two amplitudes (A) are wrong
  - -9: Three amplitudes (A) are wrong

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Solution:

(a)

The indices that most closely correspond to the frequency of the notes are the indices of the peaks. From the graph, we can see the three peak values occur at

$$k = 35, 56, 84$$

(b)

Recall digital frequencies are related to the indices of a DFT via $\omega = \frac{2\pi}{N}k$. Thus,

$$\omega_1 = \frac{2\pi}{256} \cdot 35 \approx 0.859, \quad \omega_2 = \frac{2\pi}{256} \cdot 56 \approx 1.374, \quad \omega_3 = \frac{2\pi}{256} \cdot 84 \approx 2.062$$

(c)

Recall the relation between analog and digital frequencies,

$$\omega = \Omega T \quad \Rightarrow \quad \Omega = \frac{\omega}{T} = \omega f_s$$

Thus, given $f_s = 1.2$ kHz we arrive at

$$\Omega_1 \approx 1030.835, \quad \Omega_2 \approx 1649.336, \quad \Omega_3 \approx 2474.004$$

(d)

We can convert the analog frequencies we got from part (c) into linear frequencies via the relation $f = \frac{\Omega}{2\pi}$. Thus,

$$f_1 \approx 164.063 \text{ Hz}, \quad f_2 \approx 262.5 \text{ Hz}, \quad f_3 \approx 393.75 \text{ Hz}$$

Now by referring to the given table, we find the first frequency corresponds to E3 (Octave 3), the second corresponds to C4 (Octave 4), and the third corresponds to G4 (Octave 4)

Grading: *24 points*

- +3: (a)
  - -0: Correct
  - -1: One index is wrong
  - -2: Two indices are wrong
  - -3: Three indices are wrong
- +7: (b), (c), & (d)
  - -0: Correct
  - -4: Mistakes in 3 freqiencies, but correct conversion formula is shown
  - -7: Wrong, empty, or invalid