UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

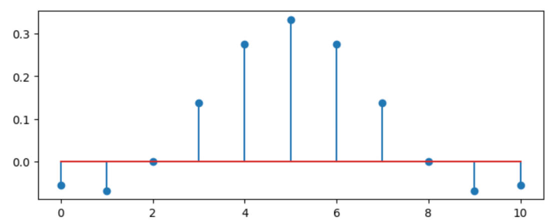
ECE 310 Digital Signal Processing – Spring 2026

Homework 12

Profs. Snyder, Shomorony **Due: Wednesday, May 6, 11:59pm on Gradescope**

Note: Only problems 1 through 6 are to be submitted. Problems 7 and 8 are only for practice.

1. The impulse response for an FIR filter is shown below. Assuming that this filter was obtained

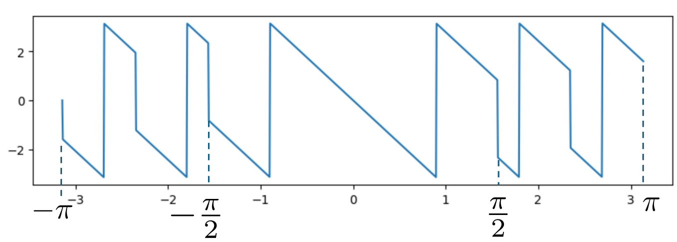


by shifting the impulse response of an ideal filter and applying a rectangular window, answer the following questions.

   (a) Is this filter GLP? If so, what type?

   (b) What kind of filter does this implement? What is the cutoff frequency?

2. An FIR filter was designed by shifting the impulse response of an ideal filter and applying a rectangular window. The phase response $\angle H_d(\omega)$ of this filter is shown below.



   (a) Is this filter strictly linear phase? Justify your answer.

   (b) What is the length of the impulse response $h[n]$?

   (c) What type of GLP filter is this?

   (d) What is $|H_d(\pi/2)|$?

3. Use the windowing method to design a length-(N) low-pass, generalized linear phase FIR filter with cut-off frequency $\pi/3$.

   (a) Find an expression for the filter coefficients $\{h[n]\}_{n=0}^{N-1}$ if the rectangular window is used for the design.

   (b) Find an expression for the filter coefficients $\{h[n]\}_{n=0}^{N-1}$ if the Hamming window is used for the design.

   (c) Comment on the pros and cons of the frequency responses of the designed filters obtained by the above two different windows.

4. Consider the below length–25 filter designed using the window method with a rectangular window.

$$h[n] = \delta[n-12] - \frac{4}{5}\text{sinc}\left(\frac{4\pi}{5}(n-12)\right), \quad 0 \le n \le 24.$$

   (a) Does this filter have generalized linear phase? If so, which of the four GLP filter types is it?

   (b) What kind of filter, i.e. lowpass, highpass, bandpass, bandstop, does $h[n]$ implement and what is/are the cutoff frequency/frequencies?

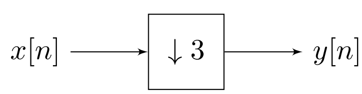
   (c) Suppose instead we would like to re-implement the original filter from $h[n]$ (i.e. same type and cutoff frequency) now at length–75 which we will call $g[n]$. Write the impulse response for $g[n]$.

   (d) Consider the below length–25 filter $f[n]$ designed using $h[n]$.

$$f[n] = 2h[n]\cos\left(\frac{\pi}{2}(n-12)\right), \quad 0 \le n \le 24.$$

   What kind of filter does $f[n]$ implement and what is/are the cutoff frequency/frequencies?

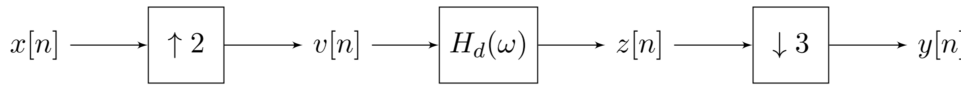
5. The below digital system has input signal $x[n] \overset{\mathcal{F}}{\longleftrightarrow} X_d(\omega)$ and output signal $y[n] \overset{\mathcal{F}}{\longleftrightarrow} Y_d(\omega)$.



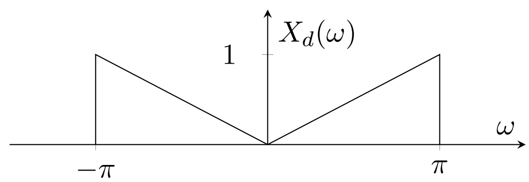
   (a) Let $x[n] = \cos\left(\frac{\pi}{6}n\right)$. Determine $y[n]$.

   (b) Let $X_d(\omega) = \dfrac{1}{1-\frac{1}{2}e^{-j\omega}}$. Show that $Y_d(\omega) = \dfrac{1}{1-\frac{1}{8}e^{-j\omega}}$.

6. Consider the below digital rate conversion system, where $H_d(\omega)$ is a low-pass filter with cutoff frequency $\omega_c = \frac{\pi}{3}$ and passband gain of 2.



The DTFT of $x[n]$, $X_d(\omega)$, is given below.

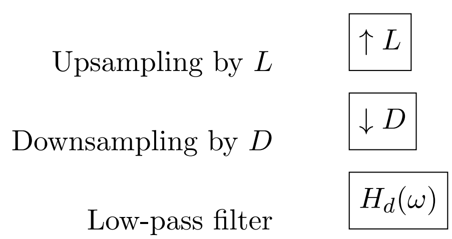


(a) Sketch the DTFT of $v[n]$, $V_d(\omega)$. Please carefully label your axes!

(b) Sketch the DTFT of $z[n]$, $Z_d(\omega)$. Please carefully label your axes!

(c) Sketch the DTFT of $y[n]$, $Y_d(\omega)$. Please carefully label your axes!

7. Your friend records a piece of music by playing an electronic piano and singing into a microphone. The piano stores data according to a sampling rate of 10 kHz while the microphone system records their voice as 24 kHz. Your friend's song is recorded for 30 seconds and if we play both signals back at their respective sampling rates they won't line up properly! Consider the below building blocks of a multirate conversion system.



We would like to synchronize the piano and voice signals, $p[n]$ and $v[n]$ at the new sampling rate of 30 kHz using the above building blocks. **For parts (b) and (c), make sure to specify the cutoff frequency and gain of each low-pass filter.**

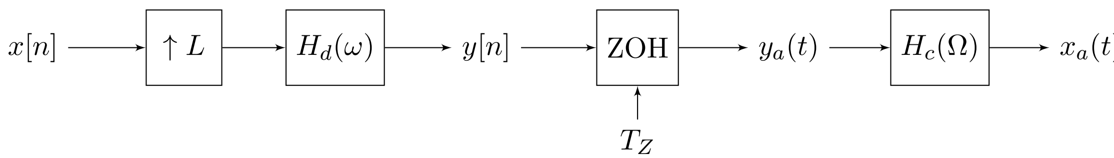
(a) Determine the number of samples in $p[n]$ and $v[n]$.

(b) Sketch a digital rate conversion system to convert $p[n]$ to sampling rate 30 kHz. Let $\hat{p}[n]$ represent to output of this system.

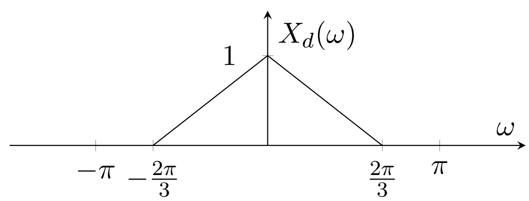
(c) Sketch a digital rate conversion system to convert $v[n]$ to sampling rate 30 kHz. Let $\hat{v}[n]$ represent the output of this system.

(d) We can properly listen to your friend's song by adding up $s[n] = \hat{p}[n] + \hat{v}[n]$. What is the length of $s[n]$?

8. Consider the following upsampled D/A with zero-order hold (ZOH) and analog compensation filter $H_c(\Omega)$. The digital filter $H_d(\omega)$ has cutoff frequency $\omega_c = \frac{\pi}{L}$ and gain L.



An analog signal $x_a(t)$ was sampled at $T = \frac{1}{200}$ s to produce $x[n]$ with the below DTFT $X_d(\omega)$.



(a) Let $L = 1$.
   i. Sketch $Y_d(\omega)$.
   ii. Determine the necessary sampling period T_Z for the ZOH.
   iii. Determine the transition bandwidth of the compensation filter $H_c(\Omega)$.

(b) Let $L = 3$.
   i. Sketch $Y_d(\omega)$.
   ii. Determine the necessary sampling period T_Z for the ZOH.
   iii. Determine the transition bandwidth of the compensation filter $H_c(\Omega)$.