HW 3

Due on **Wednesday, December 7, 2022 at 10am**

CS 598: Computational Social Choice, Fall 2022 Version: **1**

1 (100 PTS.) Nash Welfare Maximization

Prove that any allocation X that maximizes $\prod_{i \in [n]} u_i(X_i)$ is EF1 and PO, i.e., for all pairs of agents i and i', we have $u_i(X_i) \geq u_i(X_{i'} \setminus g)$ for all $g \in X_{i'}$, and there exists no other allocation Y such that $u_i(Y_i) \geq u_i(X_i)$ with at least one strict inequality.

2 (100 PTS.) Single Transferable Vote

STV is a commonly used voting scheme (e.g. in the Oscars). Recall that given a list of voters with preferences over a list of candidates, the *plurality* score of candidate c is the number of voters that has c as their top candidate.

In a STV, in each round, we eliminate the candidate with lowest plurality score and transfer its supporters votes and update the preferences appropriately (simply drop this candidate from evey preference order). The last remaining candidate is the winner of the election.

Prove that STV may not satisfy *monotonicity*: increasing the supoort for a candidate can make it worse off, i.e., show that there exists preferences $\mathcal{P} = (\succeq_1, \succeq_2, \ldots, \succeq_n)$, where the winner according to STV is a candidate a. However, there exists some preferences $\mathcal{P}' = (\succeq_1', \succeq_2', \ldots, \succeq_n')$, such that $a \succeq_i b$ implies $a \succeq_i' b$ for all $i \in [n]$, the winner according to STV is not a.

.