HW 2

Due on **Wednesday, November 30, 2022 at 10am**

CS 598: Computational Social Choice, Fall 2022 Version: **1**

1 (100 PTS.) Monotone EFX Partitions

We say that a *partition* $X = (X_1, X_2, \ldots, X_n)$ is an EFX partition if there is an assignment of bundles in X to agents such that resulting allocation is EFX. Prove that if $X = (X_1, X_2, \ldots, X_n)$ is an EFX-partition, then there exists an agent $i \in [n]$, and $g \in X_i$ such that $X' = (X_1, X_2, \ldots, X_i \setminus g, X_{i+1}, \ldots, X_n)$ is also an EFX partition.

Hint: Given a partition X, define the robust-demand bundle for agent i as $argmax_{j\in[n]} max_{g\in X_j} X_j \setminus g$. Given an EFX-partition, try to find an allocation where at least one agent has his robust-demand bundle, and then remove a good from this agent's bundle, and prove that the resulting partition is also an EFX partition.

2 (100 PTS.) MMS Allocations–Ordered Instances

Recall the definition of MMS: Given an instance $\mathcal{I}$, with a set of agents $[n]$, a set of indivisible goods M, and *additive utility functions* $u_1(\cdot), \ldots, u_n(\cdot)$[1], the MMS-feasible share of agent i, MMS_i is defined as follows:

$$MMS_i = max_{X_1,\ldots,X_n \in \mathcal{P}} min_{j \in [n]} v_i(X_j),$$

where $\mathcal{P}$ is the set of all feasible partitions. We now change the instance $\mathcal{I}$ to $\mathcal{I}'$ as follows: for each agent i, set v_{ij} to the j^{th} largest value in $\langle u_{i1}, u_{i2}, \ldots, u_{im} \rangle$ and $v_i(S) = \sum_{g \in S} v_{ig}$. Observe that in this process, all agents have the same ranking of the goods. Prove that, if there exists a partition $X = (X_1, \ldots, X_n)$ in $\mathcal{I}'$ such that $v_i(X_i) \geq \alpha MMS_i$, then there exists a partition $Y = (Y_1, \ldots, Y_n)$ such that $u_i(Y_i) \geq \alpha MMS_i$ in $\mathcal{I}$, i.e., it suffices to only consider ordered instances for proving MMS guarantees.

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[1] $u_i(S) = \sum_{g \in S} u_{ig}$

3 (100 PTS.) MMS and EF1

Prove that (i) an EF1 allocation gives $1/n$-approximate MMS allocation, and (ii) an MMS allocation does not give any approximation guarantees on an EF1 allocation.