CS 580: Algorithmic Game Theory, Fall 2026
HW 2 (due on Wednesday, 14th October at 11:59pm CST)

Instructions:

1. We will grade this assignment out of a total of 30 points.
2. You can work on any homework in groups of ($\leq$) three. Submit only one assignment per group. First submit your solutions on Gradescope, and you can add your group member after submission.
3. If you discuss a problem with another group then write the names of the other group's members at the beginning of the answer for that problem.
4. Please type your solutions if possible in Latex or doc whichever is suitable, and submit on Gradescope.
5. Even if you are not able to solve a problem completely, do submit whatever you have. Partial proofs, high-level ideas, examples, and so on.
6. You may refer to lecture slides/notes without citing. If you use any other source, including LLMs, make sure you **cite them appropriately**, and make sure to **write in your own words**.
7. No late assignments will be accepted.
8. By AGT book we mean the following book: Algorithmic Game Theory (edited) by Nisan, Roughgarden, Tardos and Vazirani. Its free online version is available at Prof. Vijay V. Vazirani's webpage.

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1. (*Recommendation: Do not use LLMs.*)

   (a) (2 points) Compute all Nash equilibria of the game shown in Table 1.

-1,4	1,-8	10, -2	3,2
3,-5	5,-2	-10, -9	5,-4
-3,-2	4,-5	-3, -5	8,-4
-2,1	4,1	9, -5	4,0

Table 1: Payoff bimatrix of a 4X4 game

[Hint: First, apply iterated dominance – iteratively remove the dominated strategies.]

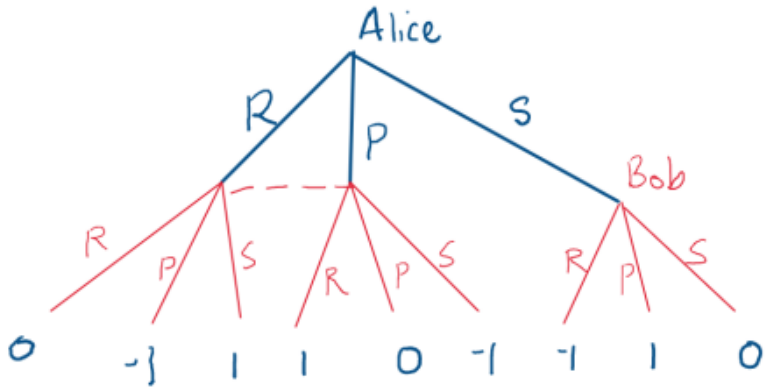

Figure 1: Extensive Form Game

(b) (3 points) Given a game (A, B), show that each of its correlated equilibria is also a coarse correlated equilibrium.

(c) (3 points) Consider the following Stackelberg game with three firms. Firm 1 chooses the quantity of its production first, then firms 2 and 3 choose their quantities simultaneously after observing firm 1's quantities. Suppose that they produce the same product with different cost functions. Firm 1's total cost is $C_1(q_1) = 10q_1 + 10$, firm 2's total cost is $C_2(q_2) = 8q_2$, and firm 3's total cost is $C_3(q_3) = 4q_3$. The firms produce identical goods and the market price is $P(q_1, q_2, q_3) = 300 - q_1 - q_2 - q_3$. What quantities do the firms produce in the subgame perfect equilibrium, when each firm is trying to maximize its profit (revenue - cost)?

(d) (2 points) Write the normal form representation of the extensive form game with imperfect information shown in Figure 1.

2. (*Recomendation: Use LLMs.*)

Given a two-player game $(A, B) \in \mathbb{R}^{m\times n} \times \mathbb{R}^{m\times n}$ with each payoff entry from $[0, 1]$, a strategy profile $(x, y) \in \mathbb{R}^{m+n}$ is said to be at ϵ-Nash equilibrium, for $\epsilon \in [0, 1]$ iff

$$x^T Ay \geq z^T Ay - \epsilon, \ \forall z \in \Delta_m \qquad \text{and} \qquad x^T By \geq x^T Bz - \epsilon, \ \forall z \in \Delta_n$$

Design an algorithm to compute 1/2-approximate Nash equilibrium (NE) of a win-lose game (A, B), where every entry in the matrices A and B is either 0 or 1.

(a) In a two-player imitation game, both players have the same set of strategies, i.e., $m = n$, and is represented by matrices (A, I) where I is an identity matrix. Let (A, I) be an imitation game where A is of the form of a matrix $\{0, 1\}^{n\times n}$. Show that the support set of strategies in a Nash equilibrium of player 2, whose payoff matrix is I, is a subset of the support set of the Nash equilibrium strategy of player 1. That is, if (x, y) is a Nash equilibrium of the game (A, I), then $supp(y) \subset supp(x)$.

(b) Using the above, show that there exists a PTAS to compute an ϵ-approximate NE of imitation games.

3. (*Experiment using LLMs*)

   Create two scenarios where strategic agents have to make decisions, and ask your favorite LLM what we should expect as an outcome. Check if the outcome pertains to Nash equilibrium of the game. Or to any other equilibrium concepts we studied. Possible scenarios may include, but are not restricted to, a bargaining problem, a routing problem, a social choice (voting) problem, and a pricing problem.

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4. *Bonus question. Can use LLMs*

   Consider a combinatorial auction with n bidders and n items where each bidder i has a unit-demand valuation v_i. This means that $v_i(S) = \max_{j \in S} v_{i,j}$ for every subset S of items. We assume that $v_{i,j} > 0$ for all i, j.

   In this auction, each bidder i submits one bid $b_{i,j}$ for each item j, and each item is sold separately using a second-price single-item auction. Assume that $b_{i,j} \in (0, v_{i,j})$ for all i, j. The utility of a bidder is her value for the items won, minus her total payment. For example, if bidder i has values v_{i1} and v_{i2} for two items, and wins both items when the second-highest bids are p_1 and p_2, then her utility is $\max\{v_{i1}, v_{i2}\} - (p_1 + p_2)$. Let $G = (A, B)$ be a bipartite graph where A is the set of bidders and B is the set of items.

   - (2 points) Show that every allocation π of items to bidders that maximizes the Social Welfare $\left(\sum_i v_{i,\pi(i)}\right)$ induces a matching on G.
   - (8 points) Show that the PoA of PNE in such a game can be at most 2. Recall that the PoA is

     $$PoA = \max_{B=(b_1,\ldots,b_n):B \text{ is a NE}} \frac{OPT}{\sum_i \max_{j \in S^B(i)} v_{ij}} = \max_{B=(b_1,\ldots,b_n):B \text{ is a NE}} \frac{\sum_i v_{i,\pi(i)}}{\sum_i \max_{j \in S^B(i)} v_{ij}}$$

     where $S^B(i)$ is the set of items assigned to agent i at NE B, and π represents the optimal allocation.

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5. *Open ended. Use LLMs*

   For approximate Nash equilibrium in finite two-player games such as (A, B) where $A, B \in [0, 1]^{m \times n}$ (see Problem 2 for the definition), a polynomial-time algorithm is known to achieve $\epsilon = 1/3$ (plus a small constant). Play with LLMs to see if this can be improved. For this, first you will have to look up what exactly is known, and what may constitute an improvement :).