CS 580: Algorithmic Game Theory, Fall 2025
HW 2 (due on Wednesday, 15th October at 11:59pm CST)

Instructions:

1. We will grade this assignment out of a total of 40 points.
2. You can work on any homework in groups of ($\leq$) two. Submit only one assignment per group. First submit your solutions on Gradescope and you can add your group member after submission.
3. If you discuss a problem with another group then write the names of the other group's members at the beginning of the answer for that problem.
4. Please type your solutions if possible in Latex or doc whichever is suitable, and submit on Gradescope.
5. Even if you are not able to solve a problem completely, do submit whatever you have. Partial proofs, high-level ideas, examples, and so on.
6. Except where otherwise noted, you may refer to lecture slides/notes. You cannot refer to textbooks, handouts, or research papers that have not been listed. If you do use any approved sources, make sure you **cite them appropriately**, and make sure to **write in your own words**.
7. No late assignments will be accepted.
8. By AGT book we mean the following book: Algorithmic Game Theory (edited) by Nisan, Roughgarden, Tardos and Vazirani. Its free online version is available at Prof. Vijay V. Vazirani's webpage.

---

1. (*Nash equilibrium: existence*)
   (a) (2 points) A player never plays a *weakly dominated* strategy with strictly positive probability at a Nash equilibrium. True or False?
   If True, give a proof. If False, give a counter example.
   (b) (3 points) Find *all* Nash equilibria of the two-player game whose payoffs are given in Table 1.
   (c) (5 points) Game (A, B) is said to be symmetric if $B = A^T$. First note that, both players have the same number of moves in a symmetric game. Prove that a symmetric game always has a symmetric NE, i.e., NE $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$ such that $y = x$.
   [Hint: Modify Nash's proof]

9,6	7,6
0,20	9,6

Table 1: Payoff bimatrix of a 3X3 game

2. (*NE to LCP*)

   (a) (4 point) Is the problem of computing a Nash equilibrium scale invariant? That is, if (x, y) is a NE of game (A, B), then for $\lambda, \kappa \geq 0$ and $a, b \in \mathbb{R}$ is it also a NE of game $(\lambda A + a, \kappa B + b)$? Justify your answer.

   (b) (6 points) Using the above, prove that finding NE in game (A, B) reduces to finding a symmetric NE in a symmetric game.

   (c) (Bonus) Using the above two, show that the problem of computing a Nash equilibrium of a 2 player game (A, B) reduces to solving an LCP (linear complementarity program) which the Lemke-Howson algorithm can solve (see Slide 2 of lecture slides where $M \geq 0$ matrix).

3. (*Other game and equilibrium notions*)

   (a) (2 points) Compute all Nash equilibria of the game shown in Table 2.

-1,4	1,-8	10, -2	3,2
3,-5	5,-2	-10, -9	5,-4
-3,-2	4,-5	-3, -5	8,-4
-2,1	4,1	9, -5	4,0

Table 2: Payoff bimatrix of a 4X4 game

   [Hint: First, apply iterated dominance – iteratively remove the dominated strategies.]

   (b) (3 points) Alice and Bob are playing a game (A, B), where $A > 0, B > 0$, in rounds where in t^{th} round they update their strategies as follows, starting at uniformly random strategies $x(0)$ and $y(0)$.

$$\forall i, \qquad x_i(t) = x_i(t-1)\frac{(Ay)_i}{x^TAy}$$
$$\forall j, \qquad y_j(t) = y_j(t-1)\frac{(x^TB)_j}{x^TBy}$$

   Show that $(x(t), y(t)) = (x(t-1), y(t-1))$ if and only if $(x(t), y(t))$ is a Nash equilibrium.

   (c) (3 points) Given a game (A, B), show that each of its correlated equilibria is also a coarse correlated equilibrium.

   (d) (2 points) Write the normal form representation of the extensive form game with imperfect information shown in Figure 1.

4. (*Stackelberg strategies*) (10 points) Consider the following Stackelberg game with three firms. Firm 1 chooses the quantity of its production first, then firms 2 and 3 choose their quantities simultaneously after observing firm 1's quantities. Suppose that they produce the same product with different cost functions. Firm 1's total cost is $C_1(q_1) = 10q_1 + 10$, firm 2's total

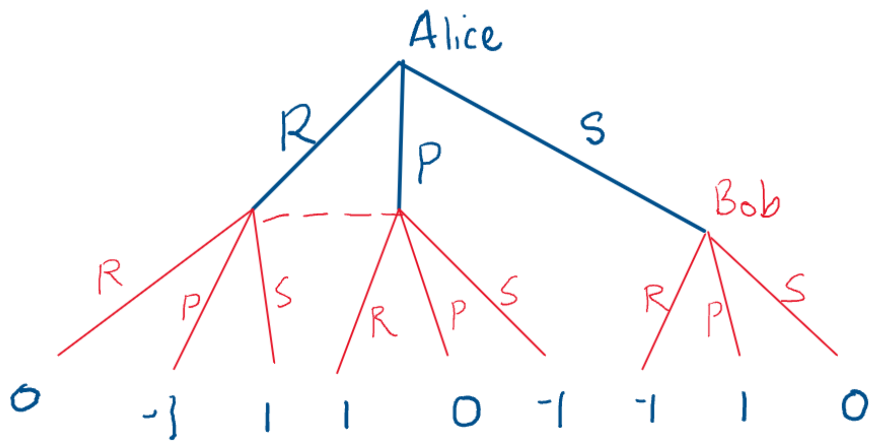

Figure 1: Extensive Form Game

cost is $C_2(q_2) = 8q_2$, and firm 3's total cost is $C_3(q_3) = 4q_3$. The firms produce identical goods and the market price is $P(q_1, q_2, q_3) = 300 - q_1 - q_2 - q_3$. What quantities do the firms produce in the subgame perfect equilibrium, when each firm is trying to maximize its profit (revenue - cost)?

5. (*Bonus problems*) Given a two-player game $(A, B) \in \mathbb{R}^{m \times n} \times \mathbb{R}^{m \times n}$ with each payoff entry from $[0, 1]$, a strategy profile $(x, y) \in \mathbb{R}^{m+n}$ is said to be at ϵ-Nash equilibrium, for $\epsilon \in [0, 1]$ iff

$$x^T Ay \geq z^T Ay - \epsilon, \ \forall z \in \Delta_m \quad \text{and} \quad x^T By \geq x^T Bz - \epsilon, \ \forall z \in \Delta_n$$

(a) Design an algorithm to compute 1/2-approximate NE equilibrium of a win-lose game (A, B), where every entry in the matrices A and B is either 0 or 1.

(b) In a two-player imitation game, both players have the same set of strategies, i.e., $m = n$, and is represented by matrices (A, I) where I is an identity matrix. Let (A, I) be an imitation game where A is of the form described in the previous sub-question. Show that the support set of strategies in a Nash equilibrium of player 2, whose payoff matrix is I, is a subset of the support set of the Nash equilibrium strategy of player 1. That is, if (x, y) is a Nash equilibrium of the game (A, I), then $supp(y) \subset supp(x)$.

(c) Using the above, show that there exists a PTAS to compute an ϵ-approximate NE of imitation games.