QUANTUM COMPLEXITY

* Complexity classes - classical and quantum
* Hamiltonian simulation

What is a complexity class?

Set of <u>decision</u> problems with similar hardness

given instance x, "decide" (Y/N) whether instance has a specific property / is in some set "L"

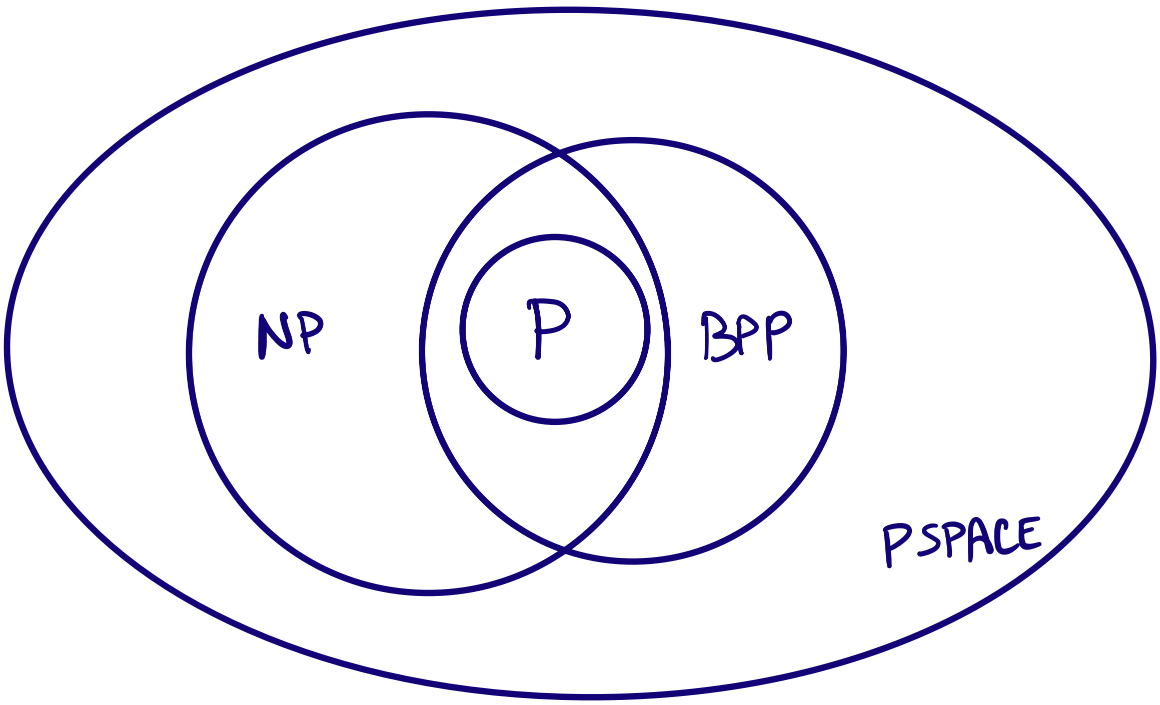
EXAMPLES

P : Solved by classical computers in (deterministic, i.e. not randomized) polynomial time

BPP : solved by randomized computers in polynomial time (with error probability $< 1/3$ on every input)

NP : for instances x in L, there is a polynomial-length "witness" that convinces a polynomial-time computer that $x \in L$.

PSPACE : solved by deterministic computer in polynomial space



Quantum analogues :

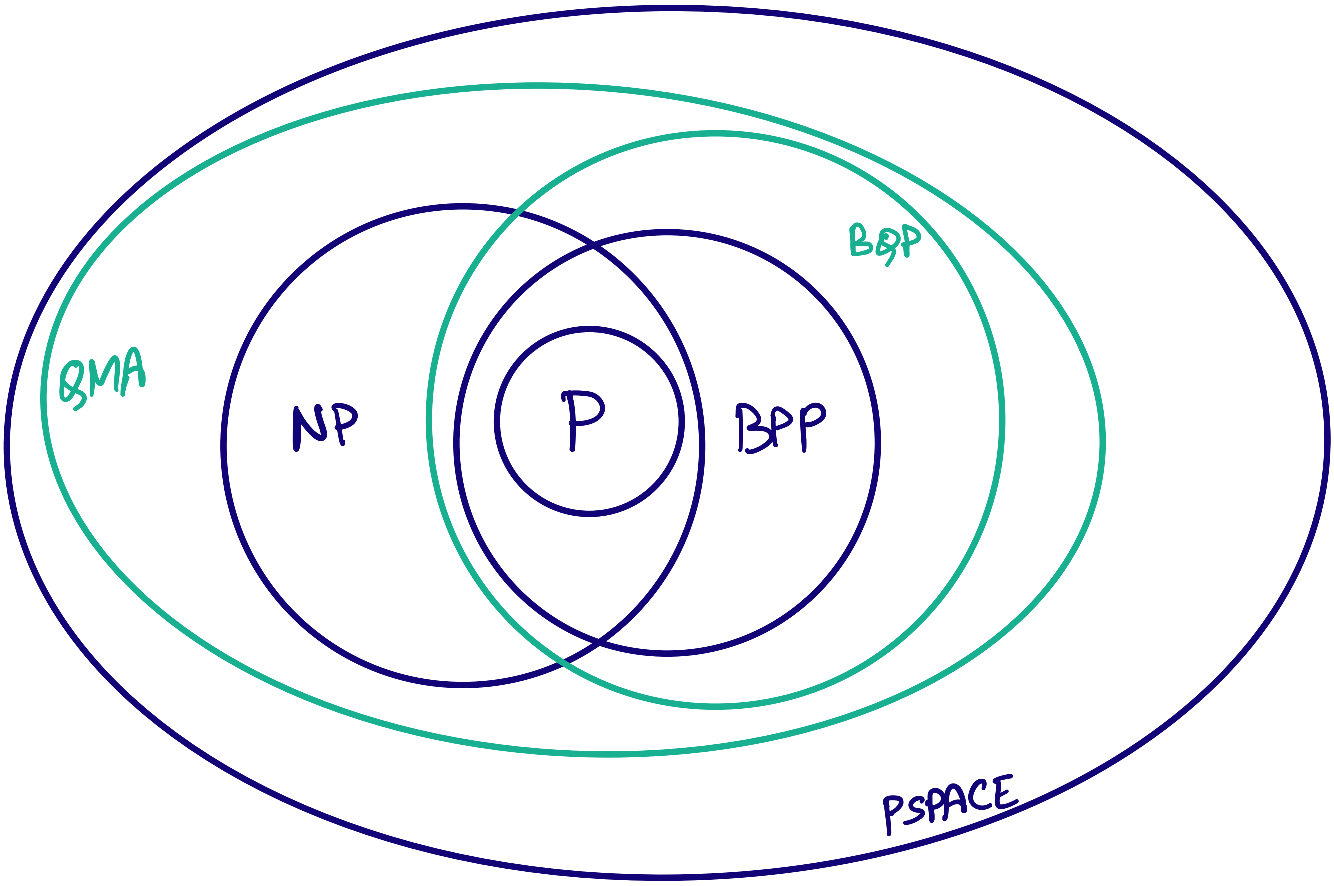
BQP : BPP, except, quantum computers

Example : factoring

"Quantum" NP ?

QMA : For every $x \in L$, there is "quantum witness" that convinces polytime verifier with error $\leq \frac{1}{3}$.

Eg : Decide if lowest eigenvalue of k-local Hamiltonian is $\leq a$ or $\geq a + \frac{1}{poly}$.



We will now take a detour into Hamiltonian complexity.

A killer app of quantum computing:

Hamiltonian simulation

evolve a quantum state over time according to a Hamiltonian

encodes information about how a collection of particles interact with each other over time

The Hamiltonian H determines which unitary will actually occur in a given physical system.

System starts at state $|\psi_0\rangle$

State at time t is $|\psi_t\rangle$, governed by

$$|\psi_t\rangle = U|\psi_0\rangle$$

where $U = e^{-iHt}$ unitary when H is Hermitian

So basically $|\psi_1\rangle = e^{-iH}|\psi_0\rangle$

$$|\psi_2\rangle = e^{-iH}|\psi_1\rangle = e^{-iH \cdot 2}|\psi_0\rangle$$

and so on...

[t need not be discrete]

Application : Quantum chemistry

figure out how a quantum system will evolve over time

many important applications

* Material sciences
* Drug discovery
* Energy industry
* Fertilizer production (eg. Nitrogen Fixation, turn nitrogen into ammonia through catalyst) 1-3% of world's energy

HAMILTONIAN SIMULATION

Input : Hamiltonian H, state $|\psi_0\rangle$ on n qubits, time t

Output : $e^{iHt}|\psi_0\rangle$

unfortunately, since H is $2^n \times 2^n$ matrix for large n, classical computers cannot efficiently compute the output. (Require time exponential in n)

What about quantum computers?

The unitary e^{iHt} is not efficiently implementable in general, for arbitrary H.

However, H that occur in nature have structure

They are typically "local": particles interact with nearby particles only

$H = H_1 + H_2 \; .... + H_m \qquad m = \text{poly}(n)$

each H_i captures "constraint" on a few qubits only

"k" - local $\Rightarrow$ Acts nontrivially on k qubits only

i.e., each $H_i = h_i \otimes I$

k-qubit matrix

for constant k, there is now a quantum algorithm with runtime $\text{poly}\left(n, m, t, \log\left(\frac{1}{\varepsilon}\right)\right)$

error