LECTURE 1, Aug 22nd, 2023

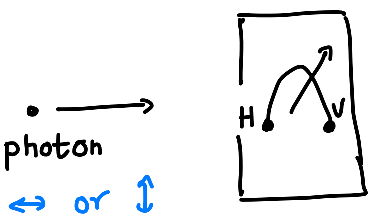
What is a qubit?

A bit : $\{0, 1\}$ ← two discrete values

How can we physically represent a bit?

Low voltage High voltage	Spin of an electron : up down	Polarization of a photon : ↔ horizontal $\lvert 0\rangle$ ↕ vertical $\lvert 1\rangle$

Only way to know the state via measuring device



photon
in ↔ or ↕ state

tells you whether photon is ↔ or ↕

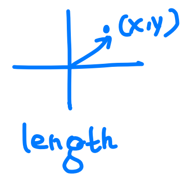
QM Law 1 If a "particle" can be in one of 2 basic states $|0\rangle$ or $|1\rangle$ then it can also be in a superposition state, meaning

"α amplitude on $|0\rangle$, β amplitude on $|1\rangle$"
where α, β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$

Recall

$z = x + iy$ complex number
where $i = \sqrt{-1}$

$|z| = \sqrt{x^2+y^2}$



Simplest quantum system with two degrees of freedom

Such a state is called a qubit.

We can represent it by a vector $\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$ ← unit vector since $|\alpha|^2 + |\beta|^2 = 1$

E.g. a photon may have the state "$\frac{1}{\sqrt{2}}$ amplitude on $|0\rangle$, $\frac{1}{\sqrt{2}}$ amplitude on $|1\rangle$" $\begin{pmatrix}1/\sqrt{2} \\ 1/\sqrt{2}\end{pmatrix}$

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

OR

"$\frac{i}{\sqrt{2}}$ amplitude on $|0\rangle$, $-\frac{1}{\sqrt{2}}$ amplitude on $|1\rangle$" $\begin{pmatrix}i/\sqrt{2} \\ -1/\sqrt{2}\end{pmatrix}$

$i = \sqrt{-1}$

$\left|\frac{i}{\sqrt{2}}\right|^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

OR

$\underbrace{\text{"1 amplitude on } |0\rangle\text{, 0 amplitude on } |1\rangle\text{"}}_{\text{called "}|0\rangle\text{"}}$ $\begin{pmatrix}1 \\ 0\end{pmatrix}$

NEXT LECTURE

You cannot read a quantum state, i.e., access α, β directly
Only way to extract information is via measurement