Multi-dimensional Arrays

Allocation and Layout of Arrays

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Importance of Array Layout

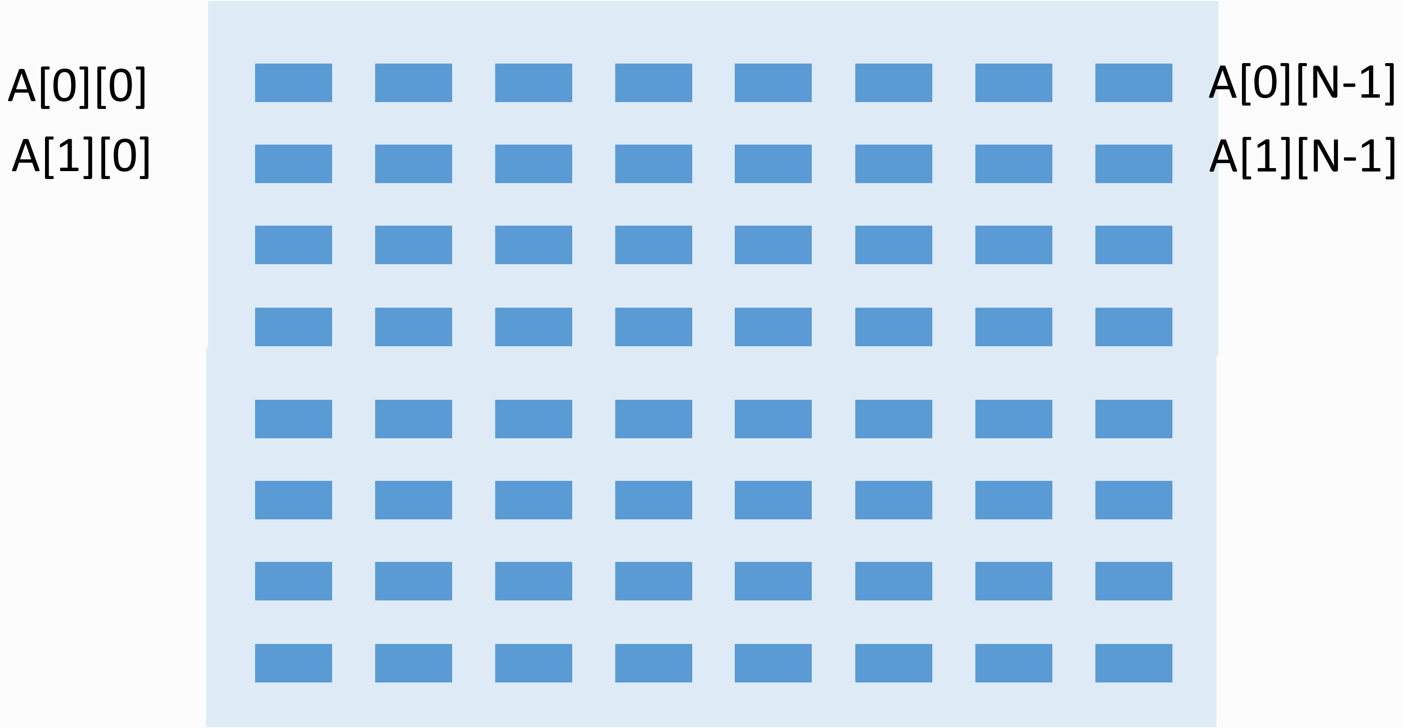
- We now know that trying to enhance spatial locality in our memory accesses is important for performance
- The reasons are somewhat circular
  - Architects observed that programmers tend to access nearby locations: e.g., linear sweep through a 1-dimensional (1-D) array
  - Provided features in hardware that improve performance for such accesses
    - Cache lines contain 64 contiguous bytes
    - Hardware prefetcher
- This is the reason why it is important to know how arrays are laid out in memory
  - Of course, a 1-D array is laid out as expected

Statically Allocated Multi-dimensional Arrays

- These are either
  - Global variables or
  - Declared inside a function (and so allocated on stack)
- int A[10][50];
- float B[e1][e2][e3];
  - Where e1,e2,e3 are expressions made of constants
  - C (C99 onwards) allows these expressions to contain variables, such as those passed as parameters of functions
- Layout for statically allocated 2-D arrays in C and C++ is "row major"
  - A[i][j+1] is adjacent in memory to A[i][j]

Layout and Cache Lines

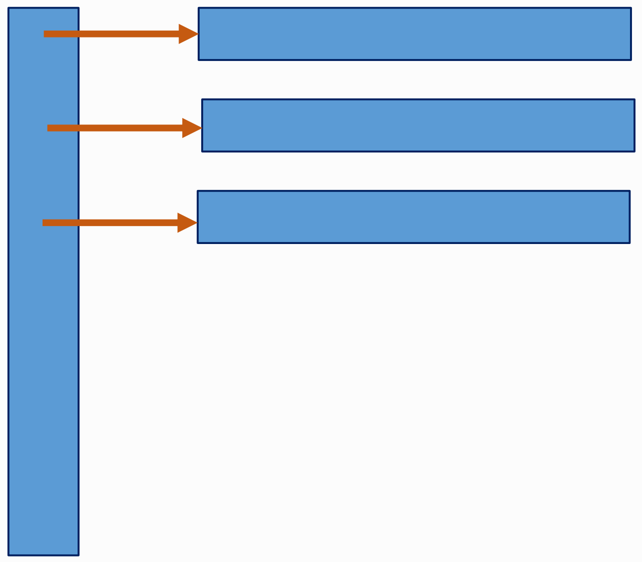
- With row-major layout, locations within a row, in consecutive columns are next to each other in memory



Dynamically Allocated Multi-dimensional Arrays

- A common method for allocating these is to create arrays of array-pointers
- But this is bad for locality
- Consecutive rows may be arbitrarily separated in memory
- Padding in allocation wastes memory
- Reduced predictability for loop accesses means prefetchers do not perform well

```
A= … malloc(sizeof(float*)*M);
for (i = 0; i < M; i++)
  A[i] = … malloc(sizeof(float) * N);
```



Dynamically Allocated Multi-dimensional Arrays II

- A better, and more general, method is to allocate all the space with one allocation call
- And then to index calculation explicitly
- Indexing is somewhat awkward, but you get used to it
  - You can use macros: indexOf(i,j,N)
  - The apparently complicated index calculations are not expensive, because the compiler can easily optimize them
- Or use an array of pointers into a contiguously allocated space

```
A= … malloc(sizeof(float*)*M*N);
   …
Instead of A[i][j], use: A[N*i + j]
```

OR

Dynamically Allocated Multi-dimensional Arrays III

- You can use similar index expressions for higher dimensional arrays
- By convention, and to retain the pattern of static allocation, the index expressions are written so as to make the last dimension contiguous

```
A = (float *)malloc(sizeof(float) * L*M*N); // in C
A = new float[L*M*N];  // in C++
  …
Instead of A[i][j][k], use:  A[M*N*i + N*j + k]
```

Cache Optimizations

Estimating Performance with Cache Misses

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To be able to effectively program a modern multiprocessor, we have to understand what it is made up of and how it came to be the way it is today

What Determines Sequential Performance

- After the code has been compiled (so compilers are out of the picture)
- The floating point units can process arithmetic at a certain rate
- The memory system can bring data into registers at a certain rate
  - By “rate,” we mean bandwidth (i.e., bytes/second)
- Which rate decides performance?
  - The slowest one
- This is quantified in the idea of floating point (or arithmetic) intensity
  - I.e., how many double precision arithmetic operations does a given code do per word (or byte) transferred between memory and registers via “load” or “store” operations

Example Code for Estimating Performance

A loop with some data accesses:

```
for (i=0; i<N; i++)
    x += A[i];
```

If there were no cache misses:

N * 0.5 ns = 0.5 ms

With cache misses:

N * 0.5 ns + (N/8) * 50 ns =

0.5 ms + 6.25 ms = 6.75 ms

More than 10 times slower

Assumptions:

- Clock rate 2 GHz (0.5 ns period per clock cycle)
- 1 FP per cycle (note: if we had FMAD operation, it could be 2)
- N is 1,000,000
- Cache line size is 64 bytes
- A is an array of doubles
  - 8 bytes each
- Cache miss penalty: 50 ns

Arithmetic Intensity: Example

- What is arithmetic intensity for the following loops?

```
for(i=0;i<N;i++)
  x+=A[i];
```

Loop 1

```
for(i=0;i<N;i++)
  x+=A[i]*A[i];
```

Loop 2

```
for(i=0;i<N;i++)
  x+=A[i]*A[i]*A[i];
```

Loop 3

- In each iteration, there is only 1 word loaded: **A[i]**
  - Why are we not counting x?
  - Because x will be in a register. Loaded once at the beginning of the loop
- How many floating point operations per iteration?
  - Let us count "+" and "*"s separately
  - 1, 2, and 3 respectively (We don't count integer arithmetic in i++. Why?)
- So, arithmetic intensity of Loop1: 1 FP/word (or 1FP/8bytes: 0.125)
- Loop2: double, Loop 3: triple (i.e 3/8)

Improving Arithmetic Intensity: Example 2

- What is arithmetic intensity for the following loops?

```
for(i=0;i<N;i++)
  x + =A[i];
for(i=0;i<N;i++)
  s += A[i]*A[i];
```

Code 1

```
for(i=0;i<N;i++) {
  x +=A[i];
  s += A[i]*A[i];
}
```

Code 2

- Code1 does 1FP op per load
- **Code 2 does 2 FP ops per load, and accomplishes the same result**
  - Loop2 will be faster

Improving Arithmetic Intensity: Example 3

- What is arithmetic intensity for the following loops?

```
for(i=0;i<N;i++)
  x + =A[i];
for(i=0;i<N;i++)
  max = A[i]>max ? A[i] :max;
```

Code 1

```
for(i=0;i<N;i++) {
  x +=A[i];
  max = A[i]>max ? A[i] :max;
 }
```

Code 2

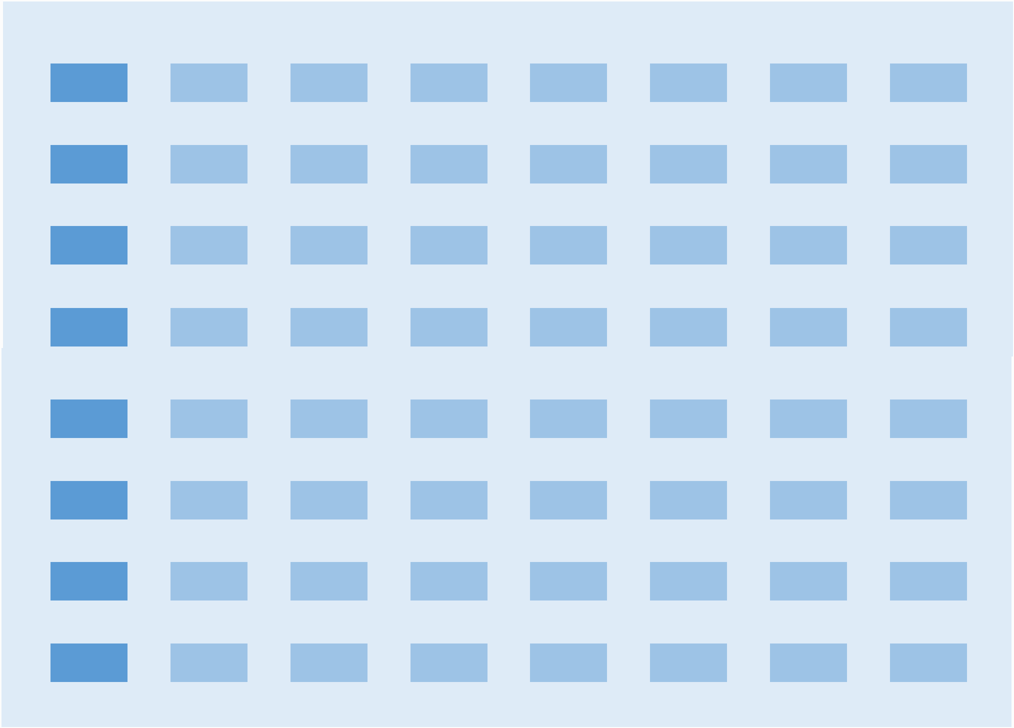
- No floating point ops in the second loop, but still code2 is better, because it incurs fewer cache misses

Cache Based Optimizations

- For a given code, with a fixed arithmetic intensity, how to improve performance?
- The basic idea is to decrease the number of cache misses

Doubly Nested Loop

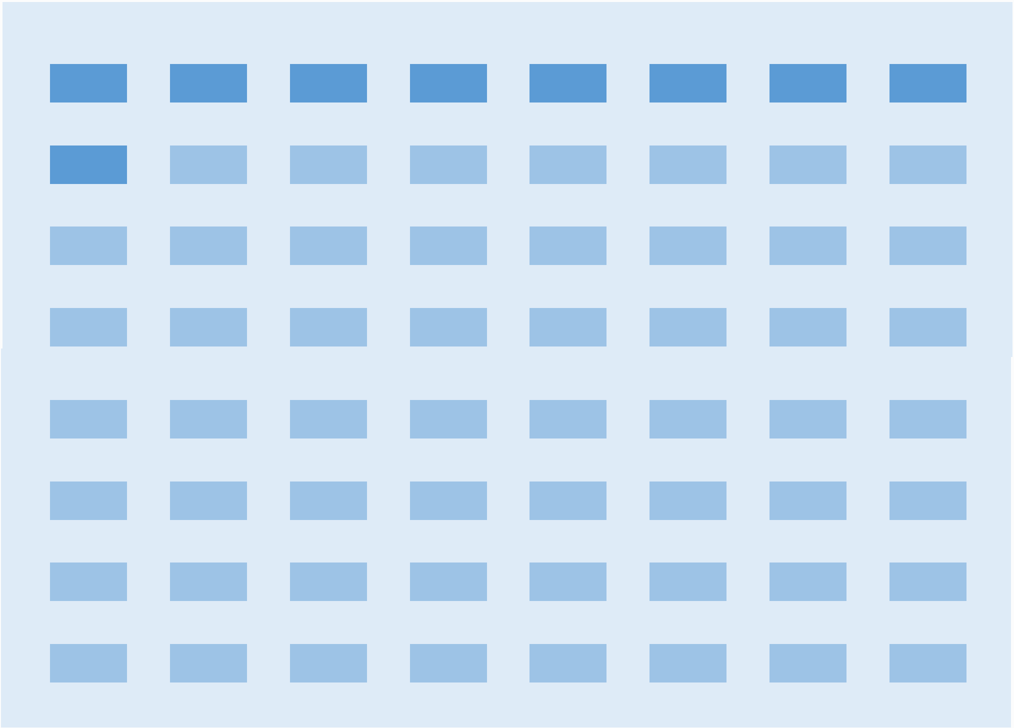
```
for(i=0;i<N;i++)
  for(j=0;j<M;j++)
    x += A[j][i];
```



- What is the problem? Count the number of misses
- Assume the cache size is less than N*w,
  - Where w is the number of words per cache line
- Every access will lead to a cache miss
  - N^2 cache misses

Fixing the Doubly Nested Loop: Reordering

```
for (j = 0; j < M; j++)
  for (i = 0; i < N; i++)
    x += A[j][i]
```



Cache Optimizations: Improving Reuse

Matrix Vector Multiplication

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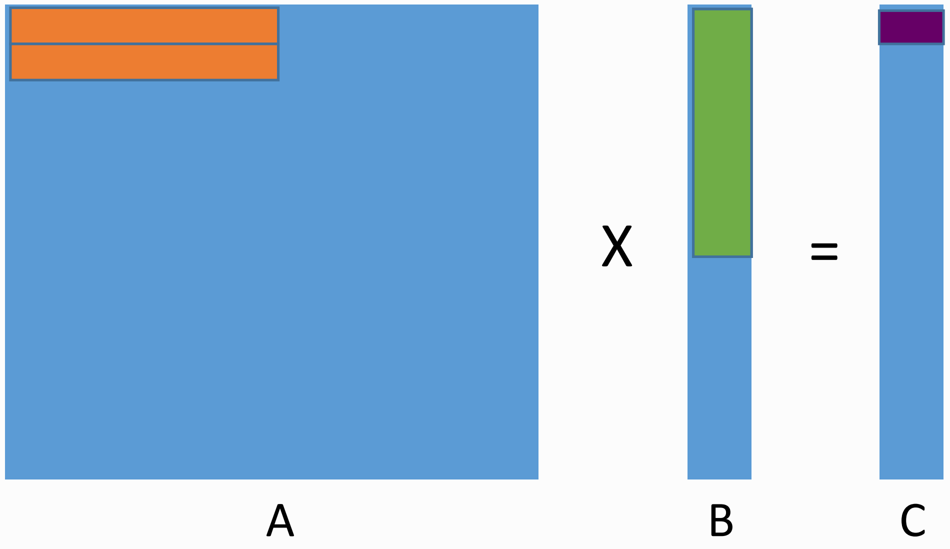
Matrix Vector Multiply

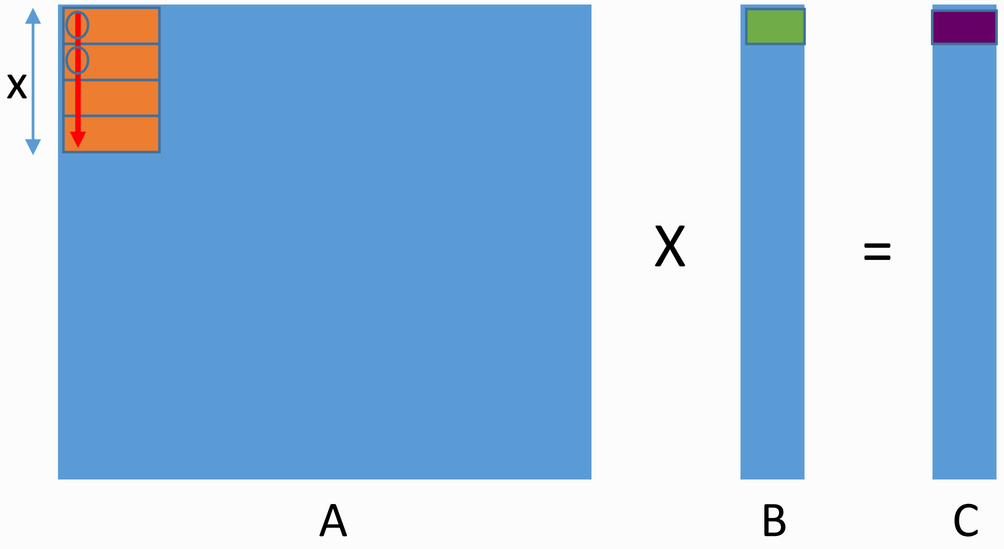
```
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    C[i] += A[i][j] * B[j]
```



- Assume cache is smaller than N words
- A and C incur only compulsory misses (N^2/w, N/w respectively)
- B is loaded multiple times, with N^2/w misses
  - For each row of A, B is traversed once, but by the time we go to the next row, the older portions of B are out of the cache

Matrix Vector Multiply: improve reuse of B?

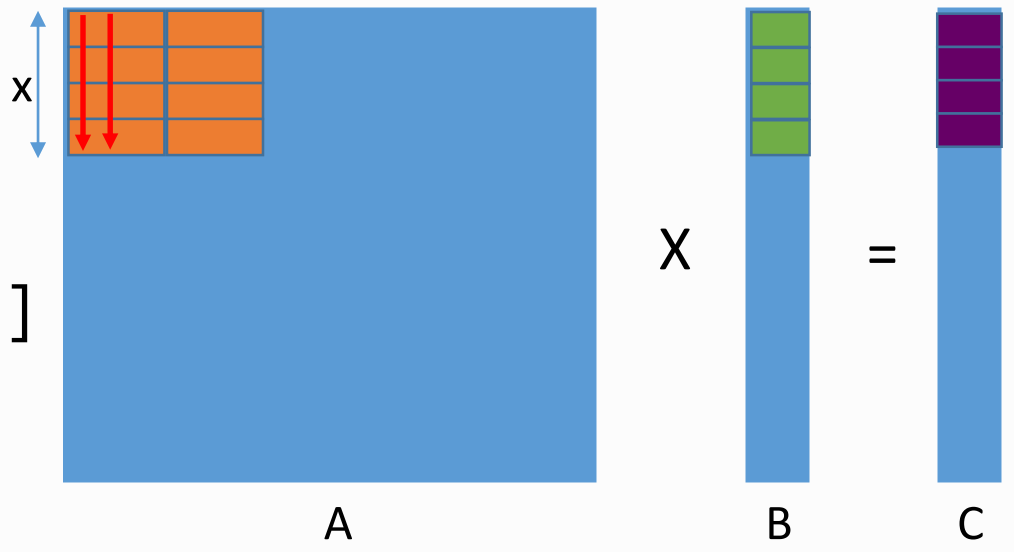
```
for (j = 0; j < N; j++)
  for(i = 0; i < N; i++)
    C[i ] += A[i ][j]*B[j]
```



- Idea: let us reuse a value from B (say B[j]) multiple times
- Lets say we load B[0]..
  - Which calculations need it?
- A loop interchange will reuse B[0], but A accesses will suffer
  - Column order traversal
- But if we do loop interchange only for X rows, the lines (orange) will still be in cache

Matrix Vector Multiply: improved

```
for (i = 0; i < N; i+= X)
  for (j = 0; j < N; j++)
    for(k = 0; k < X; k++)
      C[i+k] += A[i+k][j]*B[j]
```



- Assume cache is smaller than N words
- A and C incur only compulsory misses (N^2/w, N respectively)
- B is reused X times with total N^2/x*w misses
  - For each X rows of A, B is traversed once

Cache Optimizations: Tiling

Matrix Transpose

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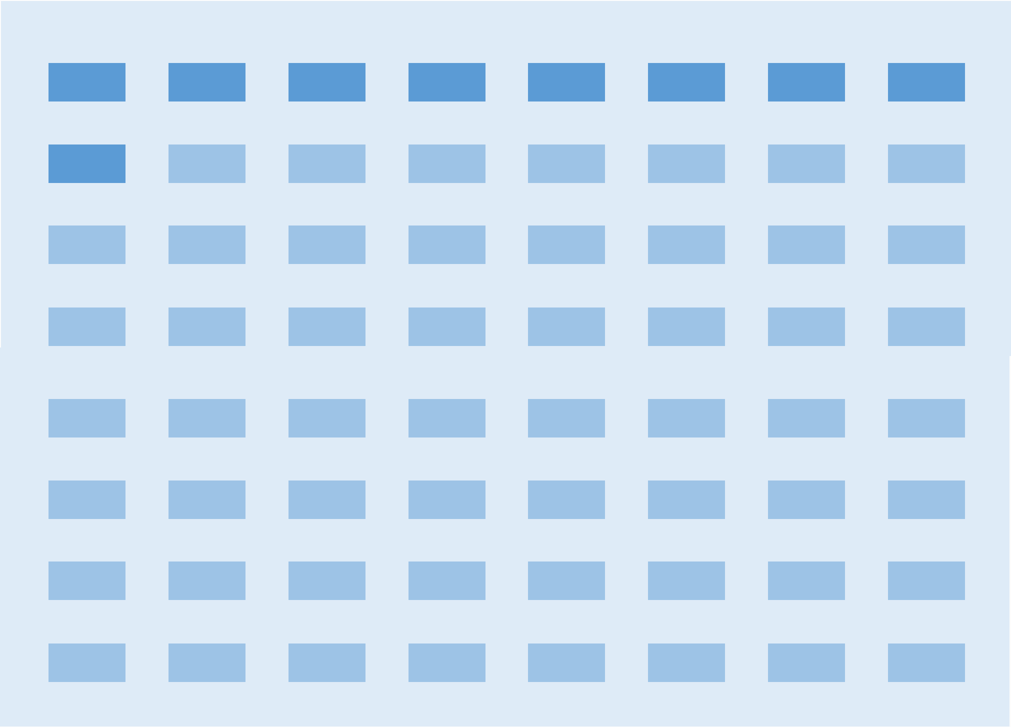
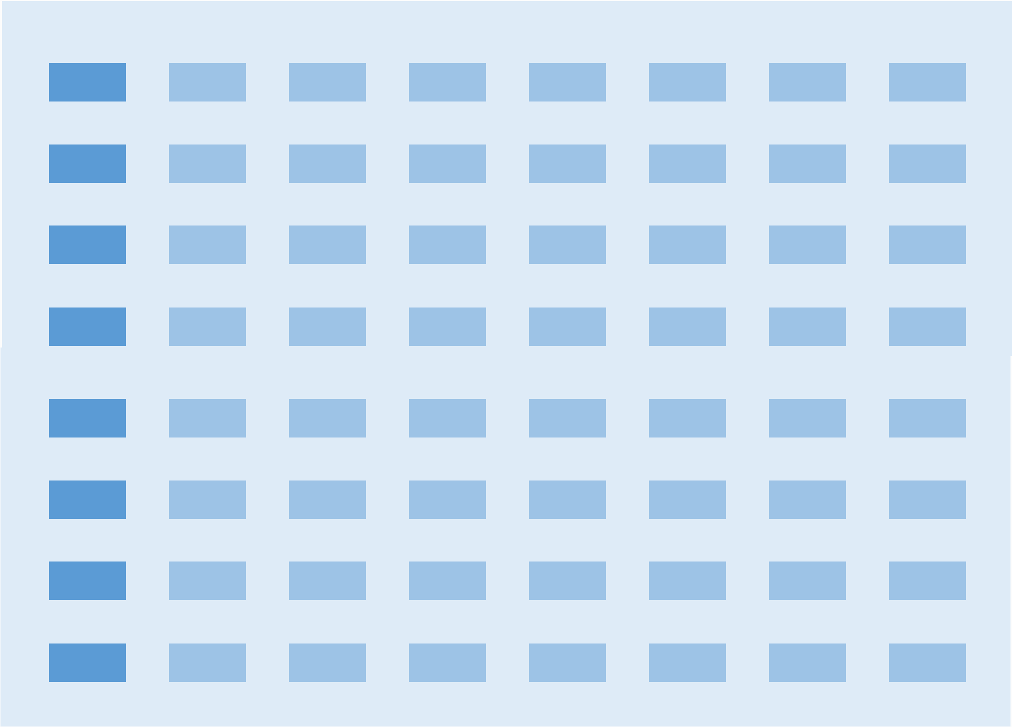
Classic Example of Optimizing for Cache Size

- Matrix transpose
- A matrix accesses: N^2 misses!
- B accesses are fine:
  - Only compulsory misses: N^2/w misses

```
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    B[i][j] = A[j][i]
```

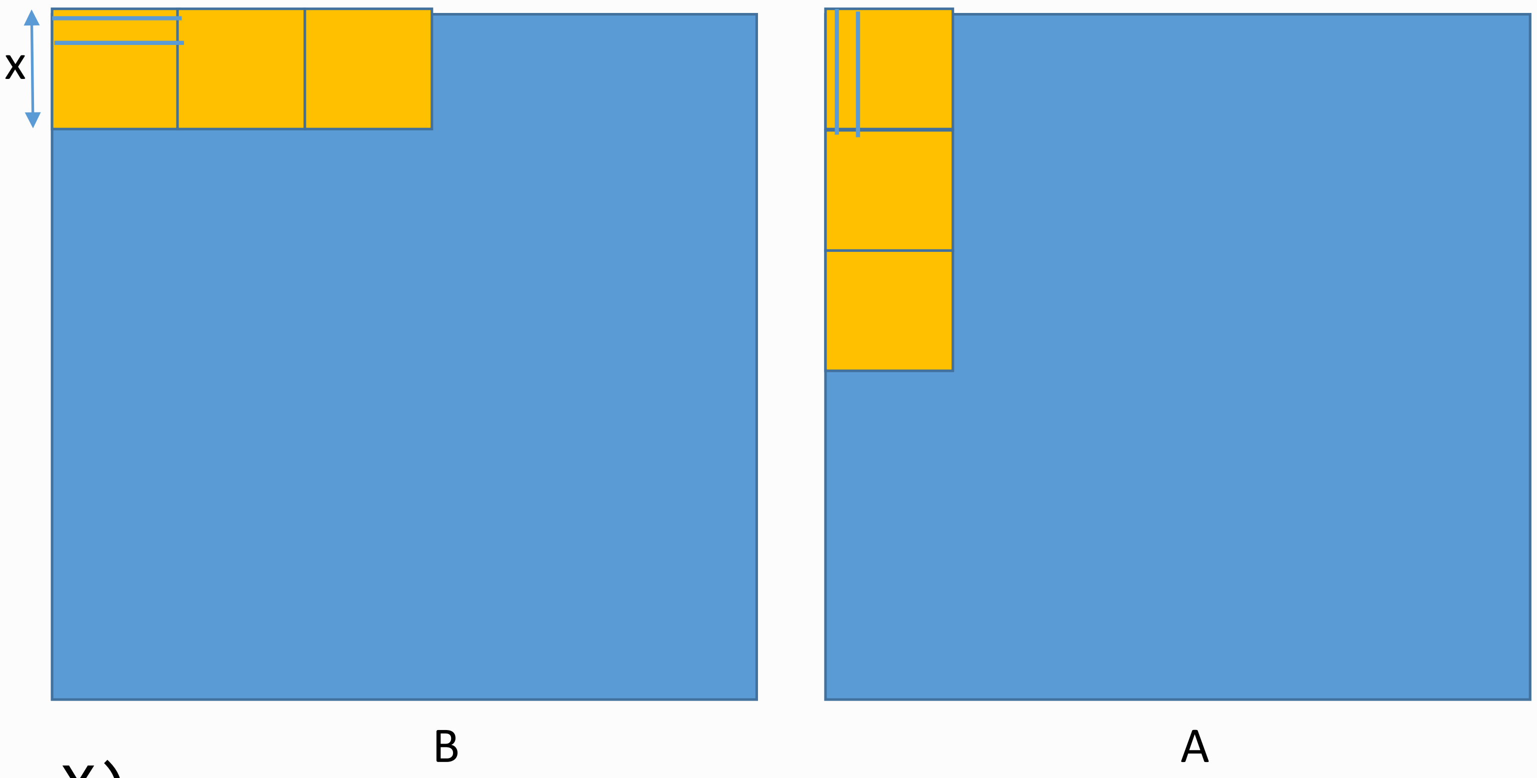


B



A

Solution: Tiling



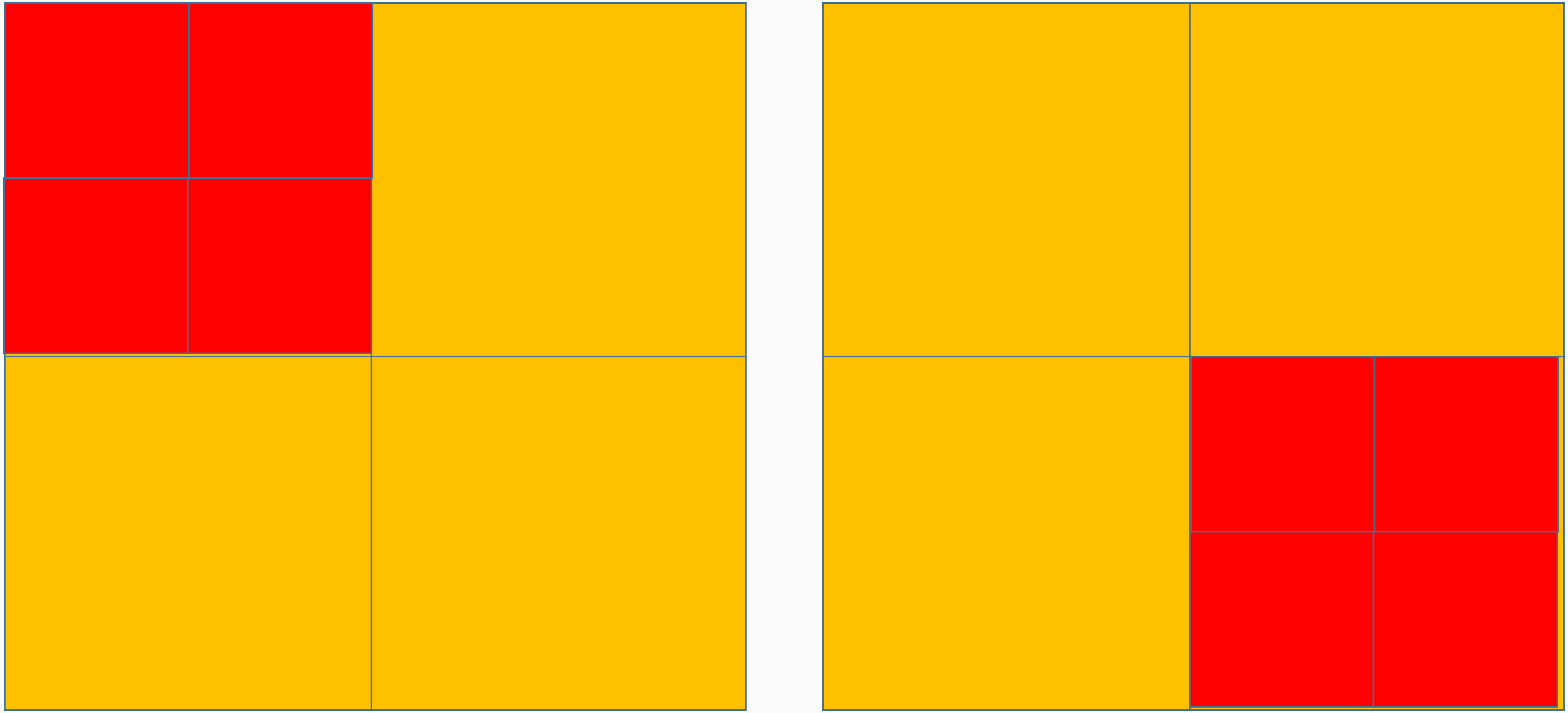
Let us assume N is a multiple of X, the tile-size.

```
for (i = 0; i < N; i += X)
  for (j = 0; j < N; j += X)
    for (p = 0; p < X; p++)
      for (q = 0; q < Y; q++)
        B[i+p][j+q] =  A[j+q][i+p]
```

Cache-Oblivious Algorithms

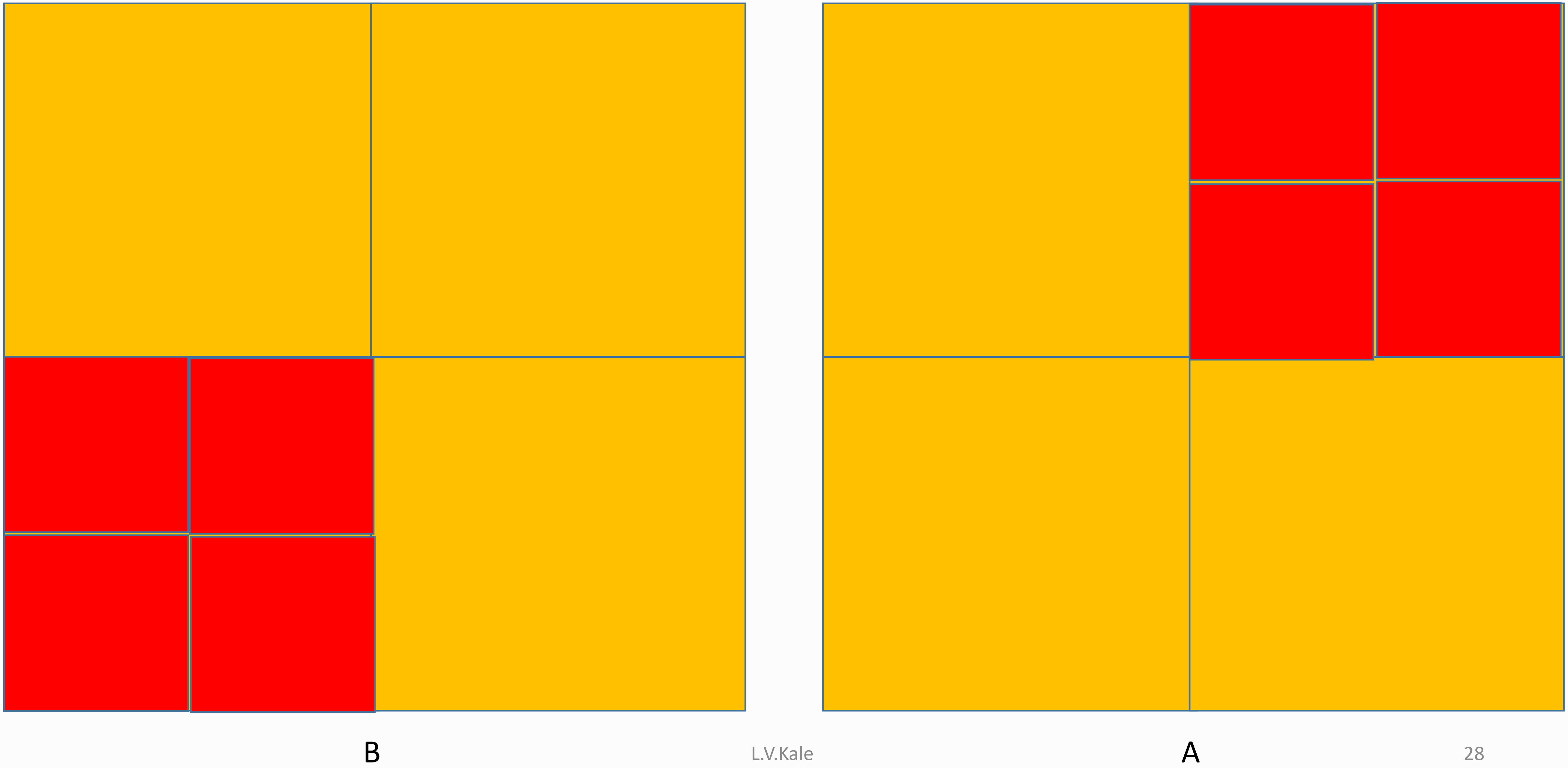
- All of the above ideas were for taking into account the specific finite cache size
- Also, we focused on only one cache level, but in reality there are L1, L2, and L3
- Another idea is to write your algorithms in a way that ignores the specific cache size, but still improves cache performance
  - Cache-oblivious algorithms, which are typically recursive

Cache-Oblivious Algorithms

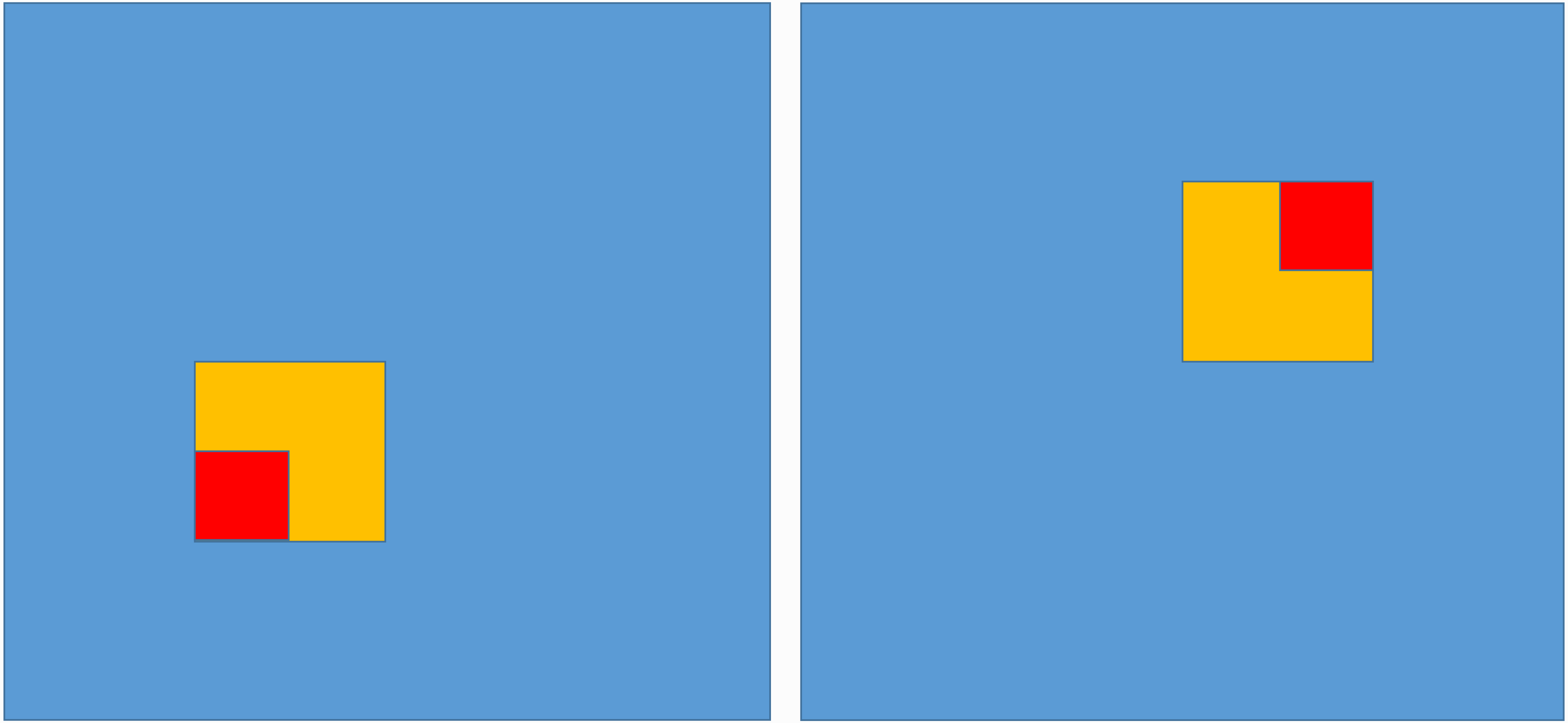


B A

Cache-Oblivious Algorithms



Cache-Oblivious Matrix Transpose



B

A

Cache-Oblivious Matrix Transpose

```
recTranspose(A, x, y, B, t, N) { // t is tilesize
  if (t < X)
    transpose(A, x, y, B, t, N);
  else {
    recTranspose(A, x,      y,      B, t/2, N);
    recTranspose(A, x,      y+t/2,  B, t/2, N);
    recTranspose(A, x+t/2,  y,      B, t/2, N);
    recTranspose(A, x+t/2,  y+t/2,  B, t/2, N);
} }
```