CS 476 Homework #10 Due 10:45am on 4/6

Note: Answers to the exercises listed below should be given in *typewritten form* (latex formatting preferred) by the deadline mentioned above. You should email your answers and also all the Maude code and all screenshots of your tool interactions for solving Problem 2 to `cs476-staff@illinois.edu`.

1. Solve Exercise 100 in *STACS*.

   Note. Your equality proofs should *only use the axioms of the theory of groups listed in Exercise 100* in *STACS* and no other equations. However, equations already proved using such axioms can also be used as *lemmas* to prove other equations.

2. Consider the following module [available in the Latex version of this homework], specifying an alternative axiomatization of the *theory of groups*, where `i` denotes the *inverse* operation $(\_)^{-1}$

```
set include BOOL off .

fmod GROUP is
  sort Group .
  op 1 : -> Group .
  op i : Group -> Group .
  op _*_ : Group Group -> Group .

  var x y z : Group .
  eq  (x * y) * z    = x * (y * z) .
  eq  1 * x          = x .
  eq  x * 1          = x .
  eq  x * i(x)       = 1 .
  eq  i(x) * x       = 1 .
  eq  i(1)           = 1 .
  eq  i(i(x))        = x .
  eq  i(x * y)       = i(y) * i(x) .
  eq  x * (i(x) * y) = y .
  eq  i(x) * (x * y) = y .
endfm
```

   Do the following:

   (a) Prove that the module `GROUP` is locally confluent by using the Church-Rosser Checker tool.

   (b) Use the MTA tool to prove the termination of `GROUP`.
   Hint. This module has a termination poof by an $A \vee C$-RPO order.

   (c) What can you now conclude from (a) and (b) above about *provable equality* in the theory of groups using *this* new axiomatization of the theory of groups instead of the axiomatization in Exercise 100 in *STACS*? Specifically, giving any equation $u = v$ in the signature of `GROUP`, can one *decide*, i.e., effectively and mechanically give a yes/no answer, to the question of wether $u = v$ can be *proved* using the equations of the theory of groups, and if so, how? Justify your answer.