ECE/CS 434 : Mobile Computing Algorithms and Applications : Homework 1 : Due 5pm, Tue, Feb 16, 2021

Problem 1 : State True/False with a 1 line justification [5x4=20 points]

(a) A is a $m \times n$ matrix with $m < n$. The null space $N(A)$ is always 0.

(b) The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, when left multiplied with a second matrix B (i.e. $B * A$), subtracts twice of the first column of B from the second column of B.

(c) For an orthonormal matrix Q (i.e., columns of Q are orthogonal to each other and the length of each column is 1), $Q^{-1} = Q^T$.

(d) If b_1, b_2, b_3 form the basis of a space, then $c_1b_1 + c_2b_2 + c_3b_3 = 0$ implies that all $c_1 = c_2 = c_3 = 0$.

(e) Matrix A has 6 columns, each column being a 10 dimensional vector. You are told that $N(A^T)$ is 5. Then, $N(A)$ must be 1 and Rank(A) must be 5.

Problem 2 : Symmetric Matrices [10+10=20 points]

(a) Prove that A^TA is a symmetric matrix.
Hint : use the basic properties of transpose, as discussed in class.

(b) Prove that $Rank(AB) \leq min\{$ Rank(A), Rank(B) $\}$

Problem 3 : Column Spaces [10+10=20 points]

(a) Choose b which gives no solution and another b which gives infinitely many solutions. Your answer should show two values of b.

$$3x + 2y = 10 \tag{1}$$
$$6x + 4y = b \tag{2}$$

(b) Consider matrix $A_{m \times n}$. You are told $r = Rank(A)$ and $r < m$ and $r < n$. How many solutions are possible for the equation $Ax = b$? What is the dimensions of $N(A)$?

Problem 4 : Least Squares [20 points]

Consider the following system of equations (called an over-determined system since there are more equations than unknowns) :

$$x - y = 2 \tag{3}$$
$$x + y = 4 \tag{4}$$
$$2x + y = 8 \tag{5}$$

How many solutions exist for the above system of equations? If a solution exists find one, if not, determine the least squares solution for x and y.

Problem 5 : Eigen values and Eigen vectors [10+10=20 points]

(a) Prove that, for symmetric matrix A, eigenvalues of matrix A^2 = (Eigenvalue of matrix A)^2

(b) Prove that $\lambda\Big(A - \sigma I\Big) = \Big(\lambda(A) - \sigma\Big)$ where $\lambda(M)$ denotes the eigenvalues of matrix M, I is the identity matrix, and σ is an arbitrary constant.