1) Consider a spherical electret of radius R with a polarization density of $\vec{P} = P_0\, \hat{z}$.

a) Find $V(r < R)$ and $V(r > R)$. $V(r > R) = \dfrac{P_0 R^3 \cos\theta}{3\varepsilon_0 r^2}$

b) Find $\vec{E}(r < R)$ and $\vec{E}(r > R)$ in spherical coordinates.

c) Find $\vec{D}(r < R)$ and $\vec{D}(r > R)$ in spherical coordinates.

d) Which components of $\vec{D}$ are continuous across r = R according to part c)? Which components of $\vec{E}$ are continuous across r = R according to part b)? What should be the correct boundary conditions?

2) A perfect conductor filling the region $z < 0$, carries a surface current of the form $\vec{K} = \beta y\,\hat{y}$ and $\vec{E} = \vec{B} = \vec{J} = 0$ within the conductor. A vacuum with no currents or charge exist in the region $z > 0$. Recall the surface boundary conditions $\vec{E}_> - \vec{E}_< = \dfrac{\sigma\hat{\eta}}{\varepsilon_0}$ and $\vec{B}_> - \vec{B}_< = \mu_0 \vec{K} \times \hat{\eta}$ where in this problem $\hat{\eta} = \hat{z}$.

a) Find $\vec{B}$ just above the surface.

b) Find the most general form of the surface charge σ using $\vec{\nabla}\Box\vec{K} + \partial\sigma/\partial t = 0$

$\sigma = -\beta t + \sigma_0$

c) Find the most general form for $\vec{E}$ just above the surface.

d) Show that your $\vec{B}$ expression of part a) and $\vec{E}$ expression of part c) satisfy each of the four Maxwell's equations.

3) A long solenoid of radius R produces a time dependent magnetic field given by $\vec{B}(s < R) = \beta\ t\ \hat{z}$.

a) Find $\vec{A}(s < R)$ and $\vec{A}(s > R)$ using $\oint \vec{A} \bullet d\ell = \int \vec{B} \bullet d\vec{a}$. $\vec{A}(s > R) = \dfrac{\beta\ t\ R^2}{2s}\hat{\phi}$

b) Verify $\left[\partial_s \vec{A}_> - \partial_s \vec{A}_<\right]_{s=R} = -\mu_0 \vec{K}$

c) Show that $\vec{E} = -\partial_t \vec{A}$ using Faraday's Law. This is easy since you just need to show that $\vec{E}$ and $-\partial_t \vec{A}$ have the same curl.

d) Find $\vec{E}\left(s < R\right)$ and $\vec{E}\left(s > R\right)$ using $\vec{E} = -\partial_t \vec{A}$

e) Check that your part d) answers satisfy $\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}$

f) Verify that your $\vec{A}(s > R)$ gives $\vec{B}\ (s > R) = 0$

1) Consider a long, conducting inner cylinder of radius *a* and length L which carries a surface current of $\vec{K}_a = K\,\hat{z}$. A coaxial log cylinder of radius b and length L carries a surface current of $\vec{K}_b$ with $b > a$. Answer all parts in terms of K, a, b, L, ω, cylindrical coordinates and physical constants as needed.

(a) Find $\vec{K}_b$ that insures that $\vec{B}(s > b) = 0$. Assume this $\vec{K}_b$ is present in all parts of this problem

(b) Calculate $\vec{B}(a < s < b)$ using Ampere's Law.

(c) Compute $\vec{A}$ in all regions using $\int \vec{A} \cdot d\vec{\ell} = \int_S \vec{B} \cdot d\vec{a}$ $\boxed{\vec{A}(a<s<b) = -\hat{z}\mu_0 aK \ln\frac{s}{a}}$

(d) Check continuity of $\vec{A}$ and discontinuity of $\frac{\partial \vec{A}}{\partial s}$ at $s = a$ and $s = b$.

We now oscillate the inner conductor current according to $\vec{K} = K \sin\omega t\,\hat{z}$.

(e) Compute $\vec{E}$ to first order in ω using Faraday's Law. Hint- the electric field is along the z-direction and the loop integral was mostly done in part (c).

(f) Check the $\vec{E}$ expression you computed in part (e) using $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

(g) Check that your $\vec{E}$ expression satisfies $\vec{E} = -\frac{\partial \vec{A}}{\partial t}$ where $\vec{A}$ is given by the "oscillating version" of the vector potential that you computed in part (c).

(h) Calculate $\vec{B}$ to second order in ω^2 by using the displacement current from the $\vec{E}$ expression you computed in part (e). This integral may be helpful

$\int \nu \ln \nu \; d\nu = \frac{\nu^2}{4}(2\ln\nu - 1)$.

2) A perfect conductor filling the region $z < 0$, carries a surface current of the form $\vec{K} = \beta y\,\hat{y}$ and $\vec{E} = \vec{B} = \vec{J} = 0$ within the conductor. A vacuum with no currents or charge exist in the region $z > 0$. Recall the surface boundary conditions $\vec{E}_> - \vec{E}_< = \frac{\sigma\hat{\eta}}{\varepsilon_0}$ and $\vec{B}_> - \vec{B}_< = \mu_0 \vec{K} \times \hat{\eta}$ where in this problem $\hat{\eta} = \hat{z}$.

a) Find $\vec{B}$ just above the surface.

b) Find the most general form of the surface charge σ using $\vec{\nabla} \cdot \vec{K} + \partial\sigma/\partial t = 0$

$\boxed{\sigma = -\beta t + \sigma_0}$

c) Find the most general form for $\vec{E}$ just above the surface.

d) Show that your $\vec{B}$ expression of part a) and $\vec{E}$ expression of part c) satisfy each of the four Maxwell's equations.

3) Consider an long solenoid with radius R and length L that holds a uniform B-field. You calculate the energy stored in the magnetic field using

$U = \frac{1}{2\mu_0}\left[\int_V \vec{B}\bullet\vec{B}\,d\tau - \int_S \left(\vec{A} \times \vec{B}\right)\bullet d\vec{a}\right]$ but instead of computing this from the volume of the solenoid, you decide to use a cylinder of length *L* and radius *a* coaxial with the solenoid for the volume term and the use surface of the cylinder for the surface. Do you get a reasonable answer? Specifically, I want you to compare the answer for the case where $a < R$ to the case where $a > R$. Why the difference?