

**Electrostatics**

$$\vec{F} = q\vec{E} \quad \vec{r} = \vec{r} - \vec{r}'$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

$$dq \rightarrow \rho d\tau' \text{ (volume)}$$

$$dq \rightarrow \sigma da' \text{ (area)}$$

$$dq \rightarrow \lambda dl' \text{ (line)}$$

$$\vec{E}(\vec{r}) = -\nabla V(\vec{r})$$

$$V(\vec{r}) = - \int_{\infty}^{\vec{r}} \vec{E}(\vec{r}) \cdot d\vec{l}$$

$$W = \frac{\epsilon_0}{2} \int |\vec{E}|^2 d\tau = \frac{1}{2} \int \rho V d\tau$$

**E-fields for simple charge configurations**

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \text{ (point charge, at origin)}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s} \hat{s} \text{ (line charge, along z axis)}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} (\pm \hat{z}) \text{ (plane charge, in xy plane)}$$

**Laplace's Equation**

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \rightarrow \nabla^2 V = 0 \text{ (if } \rho = 0)$$

**Separation of variables for  $\nabla^2 V = 0$ :**
**Cartesian:**

$$V(x, y) = \sum_{n=0}^{\infty} (A_n e^{k_n x} + B_n e^{-k_n x})(C_n \sin k_n y + D_n \cos k_n y)$$

**Spherical:**

$$V(r, \theta) = \sum_{\ell=0}^{\infty} \left( A_{\ell} r^{\ell} + \frac{B_{\ell}}{r^{\ell+1}} \right) P_{\ell}(\cos \theta)$$

**Capacitance / capacitors**

$$C = \frac{Q}{V}$$

$$C = \frac{A \epsilon_0}{d}$$

**Maxwell's Equations**

(without displacement current)

differential form:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \quad \nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{d\vec{E}}{dt}$$

integral form:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0} \quad \oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a} \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \epsilon_0 \mu_0 \int \left( \frac{d\vec{E}}{dt} \right) \cdot d\vec{a}$$

**Divergence / Stokes Theorem**

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{a} = \int_{\text{volume}} (\nabla \cdot \vec{E}) d\tau$$

$$\int_{\text{surface}} (\nabla \times \vec{B}) \cdot d\vec{a} = \oint_{\text{contour}} \vec{B} \cdot d\vec{l}$$

**Boundary conditions near sheet charge**

$$\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{n}$$

$$\frac{dV_{\text{above}}}{dn} - \frac{dV_{\text{below}}}{dn} = -\frac{\sigma}{\epsilon_0}$$

$$B_{\text{above}}^{\perp} = B_{\text{below}}^{\perp}$$

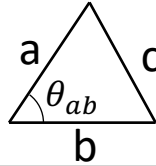
$$B_{\text{above}}^{\parallel} - B_{\text{below}}^{\parallel} = \mu_0(\vec{k} \times \hat{\ell})$$

$$\vec{A}_{\text{above}}^{\perp} = \vec{A}_{\text{below}}^{\perp}$$

$$\frac{\partial \vec{A}_{\text{above}}}{\partial n} - \frac{\partial \vec{A}_{\text{below}}}{\partial n} = -\mu_0 \vec{k}$$

**Law of cosines**

$$c^2 = a^2 + b^2 - 2ab \cos \theta_{ab}$$


**Multipole expansion**

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos \alpha) \rho(\vec{r}') d\tau'$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r} + \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2} + \dots \text{ (these are the monopole + dipole terms)}$$

$$\vec{p} \equiv \int \vec{r}' \rho(\vec{r}') d\tau'$$

**Dipoles**

$$\vec{p} = \alpha \vec{E}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$$

**First few Legendre polynomials**

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = (3x^2 - 1)/2$$

$$P_3(x) = (5x^3 - 3x)/2$$

**Polarization**

$$\vec{P} = d\vec{p}/d\tau$$

$$V_{\text{di}}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_b d\vec{a}'}{r} - \frac{1}{4\pi\epsilon_0} \int \frac{\rho_b}{r} d\tau'$$

$$\sigma_b = \vec{P} \cdot \hat{n}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

**The displacement**

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\rho = \rho_b + \rho_f$$

$$\nabla \cdot \vec{D} = \rho_f \leftrightarrow \oint \vec{D} \cdot d\vec{a}' = Q_f$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon_0(1 + \chi_e) \vec{E} = \epsilon \vec{E}$$

$$\epsilon_r = 1 + \chi_e = \epsilon/\epsilon_0$$

**Magnetostatics**

$$\vec{F} = q\vec{v} \times \vec{B} \quad \vec{r} = \vec{r} - \vec{r}'$$

$$\vec{F} = \int dl \vec{I} \times \vec{B}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{d\vec{I} \times \hat{r}}{r^2}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{d\vec{I}}{r}$$

$$d\vec{I} \rightarrow \vec{J} d\tau' \text{ (volume)}$$

$$d\vec{I} \rightarrow \vec{k} da' \text{ (area)}$$

$$d\vec{I} \rightarrow \vec{I} dl' \text{ (line)}$$

$$\vec{I} = \lambda \vec{v} \text{ (line current)}$$

$$\vec{k} = \sigma \vec{v} \text{ (sheet current)}$$

$$\vec{J} = \rho \vec{v} \text{ (volume current)}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$W = \frac{1}{2\mu_0} \int |\vec{B}|^2 d\tau = \frac{1}{2} \int (\vec{A} \cdot \vec{J}) d\tau$$

**Auxiliary field  $\vec{H}$** 

$$\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M}$$

$$\nabla \times \vec{H} = \vec{J}_f \quad \leftrightarrow \quad \oint \vec{H} \cdot d\vec{l} = I_f$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu \vec{H} \quad \mu = \mu_0(1 + \chi_m)$$

**B-fields for simple currents**

$$\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi} \quad \text{(infinite line current along z-axis)}$$

$$\vec{B} = \pm \frac{\mu_0 k}{2} \hat{y} \quad \text{(infinite sheet current in xy plane, with current in x-direction)}$$

$$\vec{B} = \mu_0 n I \hat{z} \quad \text{(infinite solenoid along z-axis)}$$

$$\vec{B} = \frac{\mu_0 N I}{2\pi s} \hat{\phi} \quad \text{(torus with radius s and symmetry axis along z)}$$

**Continuity equation, Ampere's Law for  $\vec{A}$** 

$$\nabla \cdot \vec{J} = -\frac{d\rho}{dt}$$

$$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} \quad \text{(Coulomb gauge)}$$

**Magnetization**

$$\vec{m} = I \int d\vec{a}'$$

$$\vec{\tau} = \vec{m} \times \vec{B}$$

$$\vec{M} = \frac{d\vec{m}}{d\tau}$$

$$\vec{A}_{\text{mag}}(\vec{r}) = \frac{\mu_0}{4\pi} \left[ \int \frac{\vec{J}_b(\vec{r}')}{r} d\tau' + \oint \frac{\vec{k}_b(\vec{r}')}{r} da' \right]$$

$$\vec{J}_b \equiv \nabla' \times \vec{M} \quad \text{(bound volume current)}$$

$$\vec{k}_b \equiv \vec{M} \times \hat{n} \quad \text{(bound sheet current)}$$

**Circuits**

$$R = \frac{\rho}{qB_0}$$

$$\vec{l} = \lambda \vec{v}$$

$$\vec{j} = \sigma \vec{E} \quad \text{or} \quad V = IR$$

$$P = I^2 R$$

$$\varepsilon = \oint \vec{f} \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l}$$

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

$$\varepsilon = - \frac{d\Phi}{dt}$$

$$L = \Phi / I$$

$$W_C = \frac{1}{2} QV = \frac{1}{2} QC^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$W_L = \frac{1}{2} LI^2 = \frac{1}{2} I\Phi$$

**Waves**

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{d^2 \vec{E}}{dt^2}$$

$$\nabla^2 \vec{B} = \frac{1}{c^2} \frac{d^2 \vec{B}}{dt^2}$$

$$\omega = ck$$

$$\vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \phi)$$

$$\vec{B} = \vec{B}_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \phi)$$

$$\vec{k} = \vec{E}_0 \times \vec{B}_0$$

**Maxwell's Equations in a Medium**

$$\nabla \cdot \vec{D} = \rho_b$$

$$\oint \vec{D} \cdot d\vec{a} = Q_f$$

$$\nabla \cdot \vec{B} = 0$$

$$\oint \vec{B} \cdot d\vec{a} = 0$$

$$\nabla \times \vec{E} = - \frac{d\vec{B}}{dt}$$

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

$$\nabla \times \vec{H} = \vec{j}_f + \frac{d\vec{D}}{dt}$$

$$\oint \vec{H} \cdot d\vec{l} = I_f + \int \left( \frac{d\vec{D}}{dt} \right) \cdot d\vec{a}$$

## Spherical Coordinates

Line Element:  $d\vec{l} = dr \hat{r} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{\phi}$

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}(\sqrt{x^2 + y^2} / z)$$

$$\phi = \tan^{-1}(y / x)$$

$$\hat{x} = \sin\theta \cos\phi \hat{r} + \cos\theta \cos\phi \hat{\theta} - \sin\phi \hat{\phi}$$

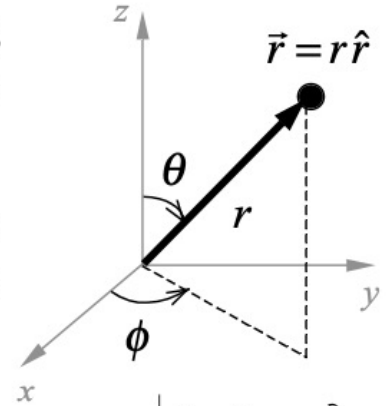
$$\hat{y} = \sin\theta \sin\phi \hat{r} + \cos\theta \sin\phi \hat{\theta} + \cos\phi \hat{\phi}$$

$$\hat{z} = \cos\theta \hat{r} - \sin\theta \hat{\theta}$$

$$\hat{r} = \sin\theta \cos\phi \hat{x} + \sin\theta \sin\phi \hat{y} + \cos\theta \hat{z}$$

$$\hat{\theta} = \cos\theta \cos\phi \hat{x} + \cos\theta \sin\phi \hat{y} - \sin\theta \hat{z}$$

$$\hat{\phi} = -\sin\phi \hat{x} + \cos\phi \hat{y}$$



Gradient:  $\vec{\nabla}V = \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \hat{\phi}$

Laplacian:  $\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$

Divergence:  $\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta E_\theta) + \frac{1}{r \sin\theta} \frac{\partial E_\phi}{\partial \phi}$

Curl:  $\vec{\nabla} \times \vec{E} = \frac{\hat{r}}{r \sin\theta} \left[ \frac{\partial}{\partial \theta} (\sin\theta E_\phi) - \frac{\partial E_\theta}{\partial \phi} \right] + \frac{\hat{\theta}}{r} \left[ \frac{1}{\sin\theta} \frac{\partial E_r}{\partial \phi} - \frac{\partial}{\partial r} (r E_\phi) \right] + \frac{\hat{\phi}}{r} \left[ \frac{\partial}{\partial r} (r E_\theta) - \frac{\partial E_r}{\partial \theta} \right]$

	$\partial_r$	$\partial_\theta$	$\partial_\phi$
$\hat{r}$	0	$\hat{\theta}$	$\sin\theta \hat{\phi}$
$\hat{\theta}$	0	$-\hat{r}$	$\cos\theta \hat{\phi}$
$\hat{\phi}$	0	0	$-\sin\theta \hat{r}$ $-\cos\theta \hat{\theta}$

## Cylindrical Coordinates

Line Element:  $d\vec{l} = ds \hat{s} + s d\phi \hat{\phi} + dz \hat{z}$

$$x = s \cos\phi$$

$$y = s \sin\phi$$

$$z = z$$

$$s = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y / x)$$

$$z = z$$

$$\hat{x} = \cos\phi \hat{s} - \sin\phi \hat{\phi}$$

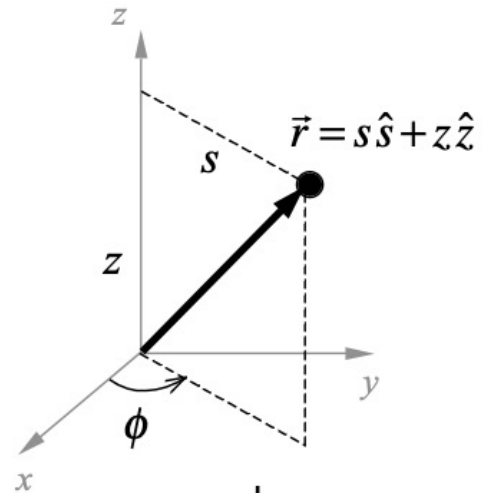
$$\hat{y} = \sin\phi \hat{s} + \cos\phi \hat{\phi}$$

$$\hat{z} = \hat{z}$$

$$\hat{s} = +\cos\phi \hat{x} + \sin\phi \hat{y}$$

$$\hat{\phi} = -\sin\phi \hat{x} + \cos\phi \hat{y}$$

$$\hat{z} = \hat{z}$$



Gradient:  $\vec{\nabla}V = \frac{\partial V}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial V}{\partial \phi} \hat{\phi} + \frac{\partial V}{\partial z} \hat{z}$

Laplacian:  $\nabla^2 V = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial V}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

Divergence:  $\vec{\nabla} \cdot \vec{E} = \frac{1}{s} \frac{\partial}{\partial s} (s E_s) + \frac{1}{s} \frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z}$

Curl:  $\vec{\nabla} \times \vec{E} = \left[ \frac{1}{s} \frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} \right] \hat{s} + \left[ \frac{\partial E_s}{\partial z} - \frac{\partial E_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[ \frac{\partial}{\partial s} (s E_\phi) - \frac{\partial E_s}{\partial \phi} \right] \hat{z}$

	$\partial_s$	$\partial_\phi$	$\partial_z$
$\hat{s}$	0	$\hat{\phi}$	0
$\hat{\phi}$	0	$-\hat{s}$	0
$\hat{z}$	0	0	0