ECE 333

Green Electric Energy

Lecture 16

Solar Position (4.4); Sun Path Diagrams and Shading (4.5); Clear-Sky Direct-Beam Radiation (4.9)

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Department of Electrical and Computer Engineering

Slides Courtesy of Prof. Tim O’Connell and George Gross

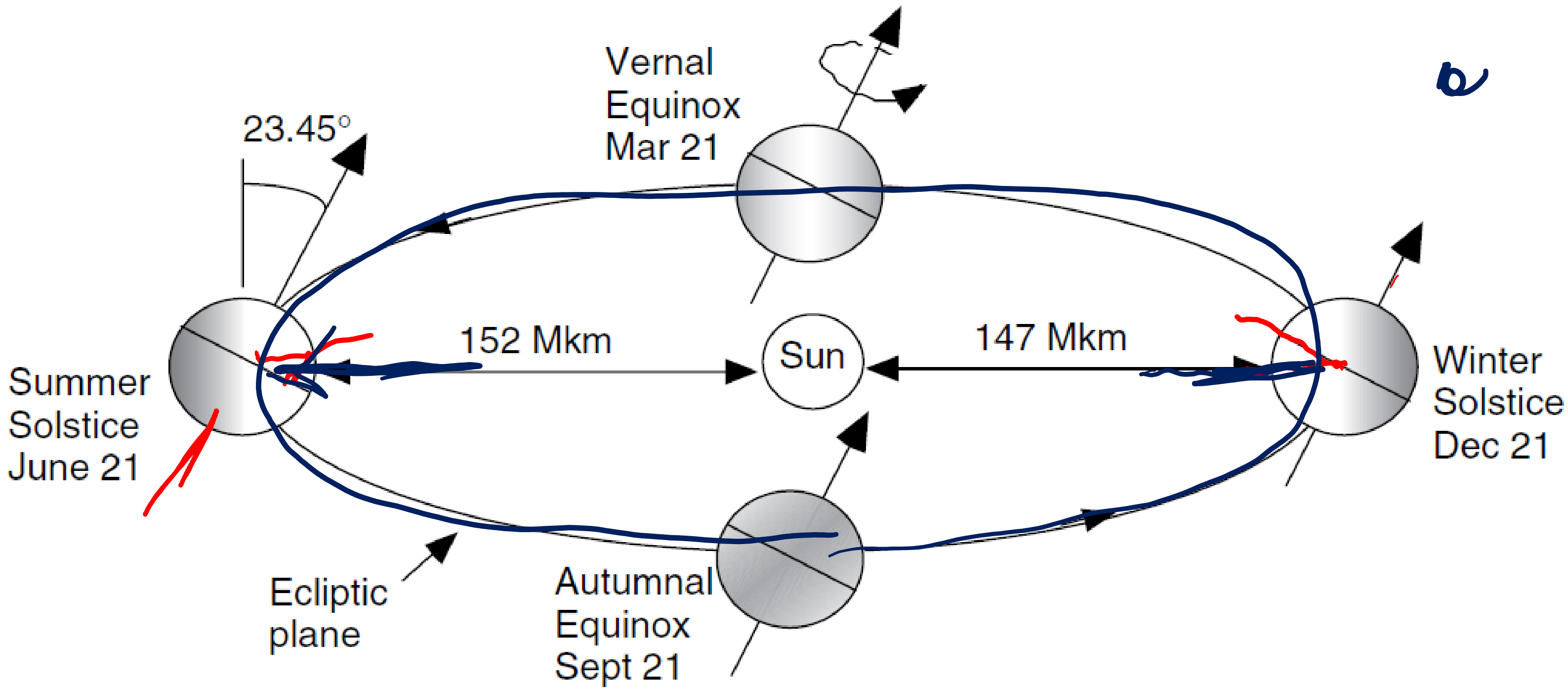
Announcements

- Announcements:
  - Exam 2 Details
- Today
  - 4.3: Review: Altitude Angle of the Sun at Solar Noon
  - 4.4: Solar Position at Any Time of Day
  - 4.5: Sun Path Diagrams for Shading Analysis
  - 4.9: Clear-Sky Direct-Beam Radiation

Exam 2 Discussion

- Date: April 9^{th}
- Format: same as Exam 1
- Time: 24-hour window on April 9^{th}
- Length: same as Exam 1
- Resources: Open book, open notes, open computer - You are welcome to use any resource ***except another person***.
- Bonus points: 5 bonus points on Exam 2 for attaching your ***handwritten*** note sheets for Exam 1 and 2 (8.5” x 11”, front and back)
  - Attach to the end of the test
- Submission: through Gradescope

The Earth's Orbit



Equinox – equal day and night, on March 21 and September 21

Winter solstice – North Pole is tilted furthest from the sun

Summer solstice – North Pole is tilted closest to the sun

4.3: Altitude Angle of the Sun at Solar Noon

- **Solar declination** δ – the angle formed between the plane of the equator and the line from the center of the sun to the center of the earth
- δ varies between +/- 23.45˚

The Sun's Position in the Sky

- Solar declination from an Earth-centric perspective
  - Note: solar declination varies over the year, not during the day

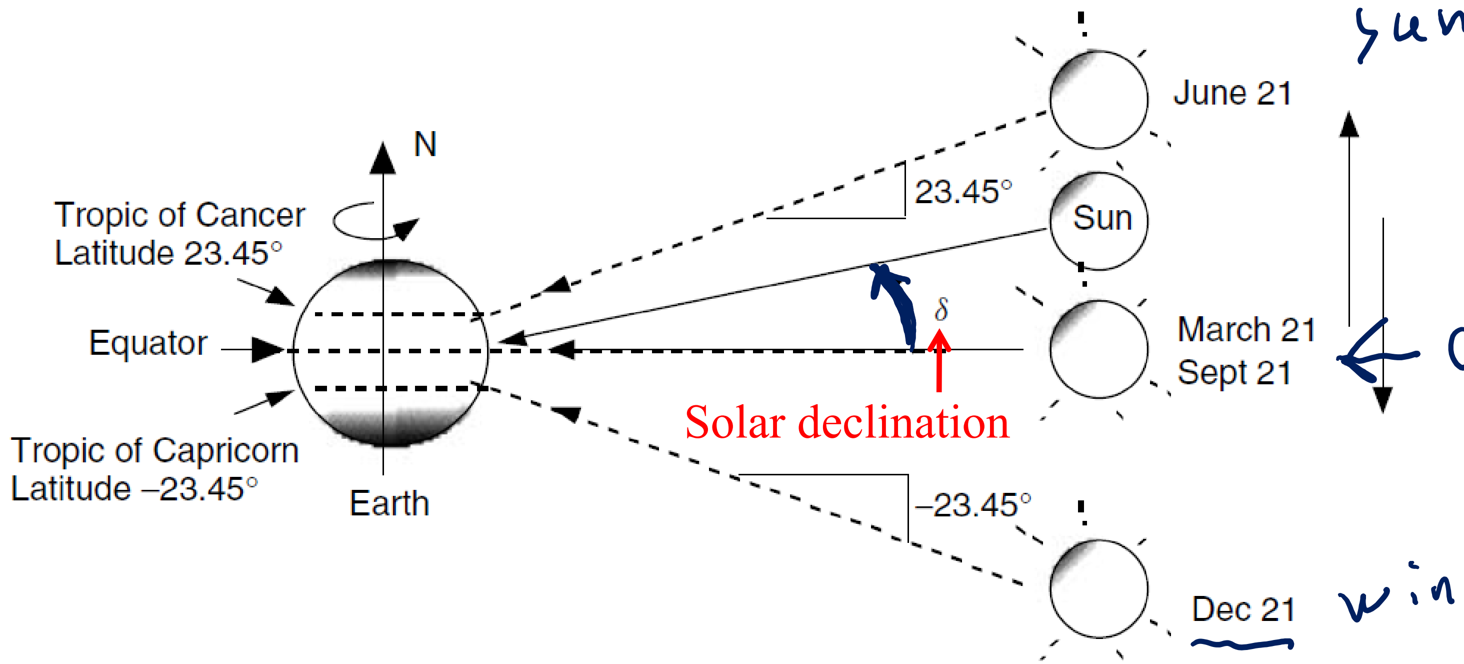


Figure 4.5 An alternative view with a fixed earth and a sun that moves up and down. The angle between the sun and the equator is called the solar declination δ.

4.3: Altitude Angle of the Sun at Solar Noon

- **Solar declination** δ – the angle formed between the plane of the equator and the line from the center of the sun to the center of the earth
- δ varies between +/- 23.45˚
- Assuming a sinusoidal relationship, a 365 day year, and n=81 is the Spring equinox, the approximation of δ for any day n can be found from

degrees

$$\delta = 23.45 \sin\left[\frac{360^\circ}{365}(n-81)\right] \quad (4.6)$$

Solar Noon and Collector Tilt

- **Solar noon** – sun is directly over the local line of longitude
- Rule of thumb for the Northern Hemisphere: a South-facing collector tilted at an angle equal to the local latitude

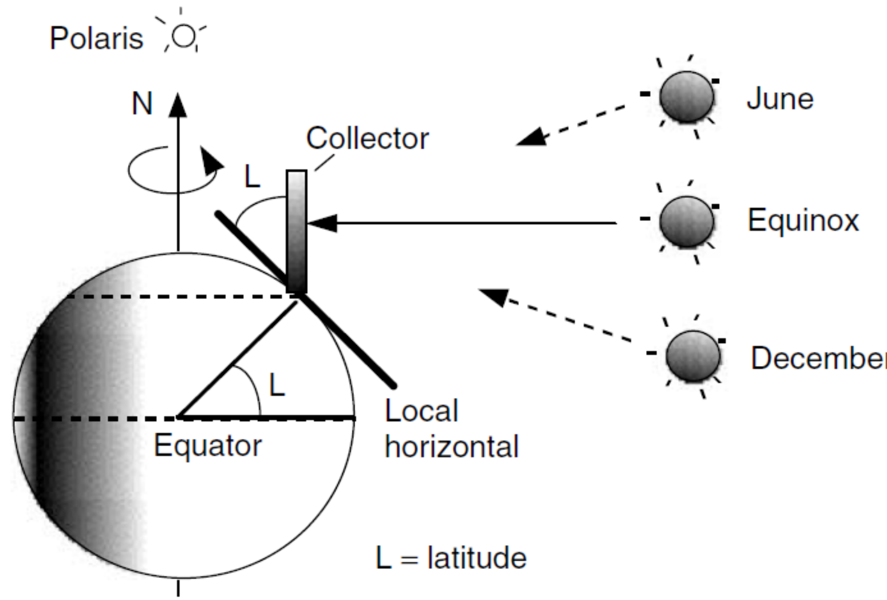


Figure 4.8 A south-facing collector tipped up to an angle equal to its latitude is perpendicular to the sun's rays at solar noon during the equinoxes.

- In this case, on an <u>equinox</u>, during solar noon, the sun's rays are perpendicular to the collector face

Altitude Angle $\boldsymbol{\beta_N}$ at Solar Noon

- Altitude angle at solar noon β_N – angle between the Sun and the local horizon

$$0 \le \beta_N \le 90^\circ$$

$$0 \le L \le 90^\circ$$

$$-23.45^\circ \le \delta \le 23.45^\circ$$

$$\beta_N = 90^\circ - L + \delta \quad (4.7)$$

- Zenith – perpendicular axis at a site

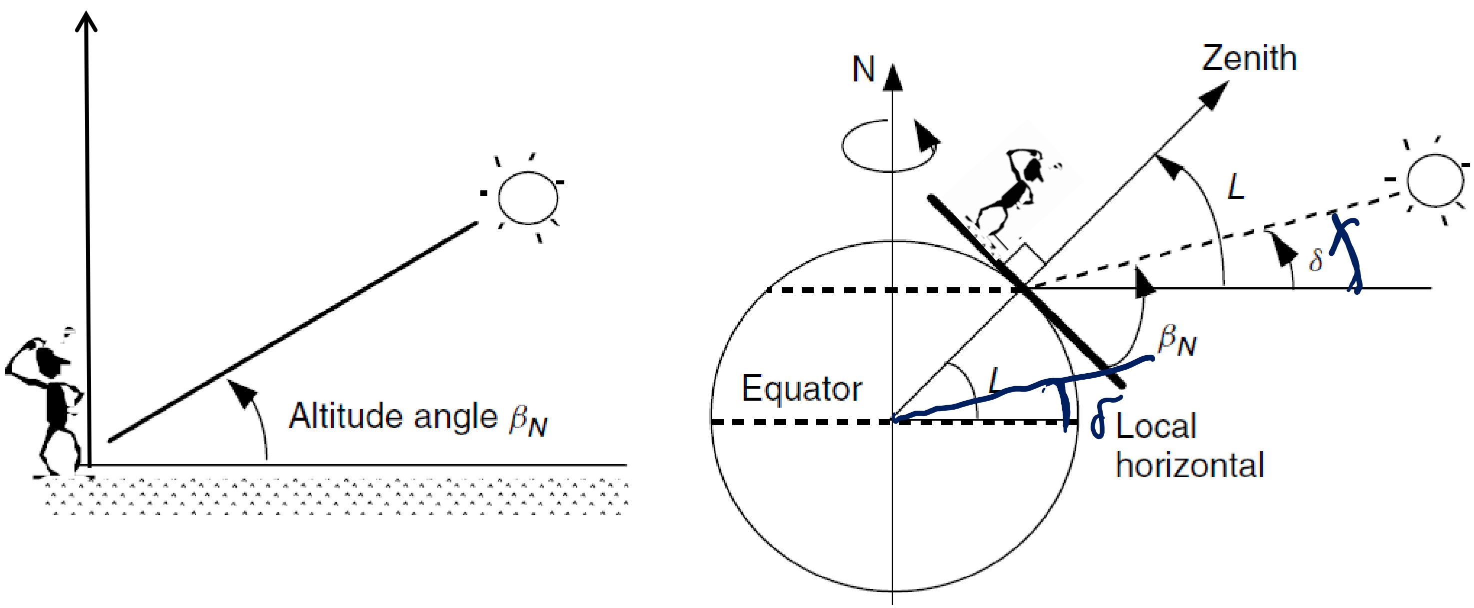
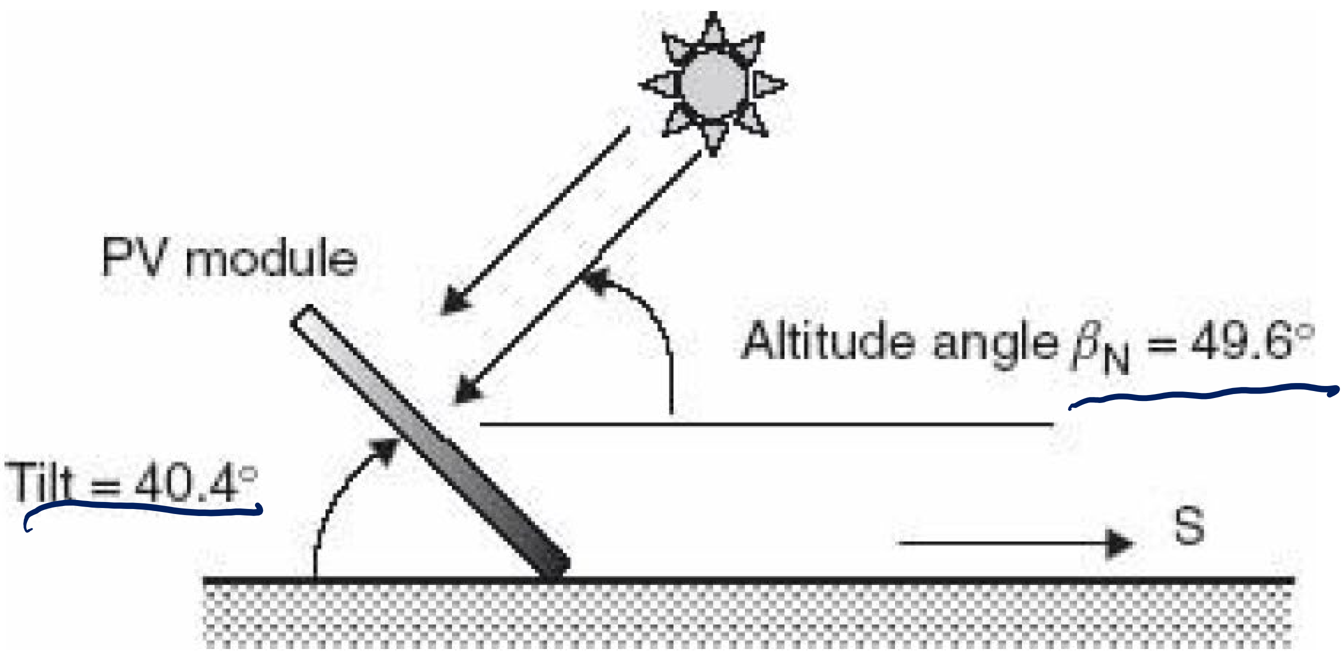


Figure 4.9 The altitude angle of the sun at solar noon.

Tilt Angle of a Photovoltaic (PV) Module

- Rule of thumb: Tilt angle = $90° - \beta_N$



Example 4.2

- **Example 4.2 Tilt Angle of a PV Module.** Find the optimum tilt angle for a south-facing photovoltaic module in Tucson (latitude 32.1°) at solar noon on March 1.

Table 4.1 → March 1 => 60th day = n

$$\delta = 23.45 \cdot \sin\left(\frac{360}{365}(n-81)\right) = -8.3°$$

$$\beta_N = 90° - L + \delta = 90 - 32.1° + (-8.3°) = 49.6°$$

$$\text{Tilt} = 90° - \beta_N = 90° - 49.6 = 40.4°$$

4.4: Solar Position at Any Time of Day

- Described in terms of **altitude angle**, β, and **azimuth angle**, ϕ_S, of the sun
- β and ϕ_S depend on latitude, day number, and time of day
- Azimuth angle (ϕ_S) convention
  - Positive in the morning when Sun is in the East
  - Negative in the evening when Sun is in the West
  - Reference in the Northern Hemisphere (for us) is true South
- Hours are referenced to solar noon

Altitude Angle and Azimuth Angle

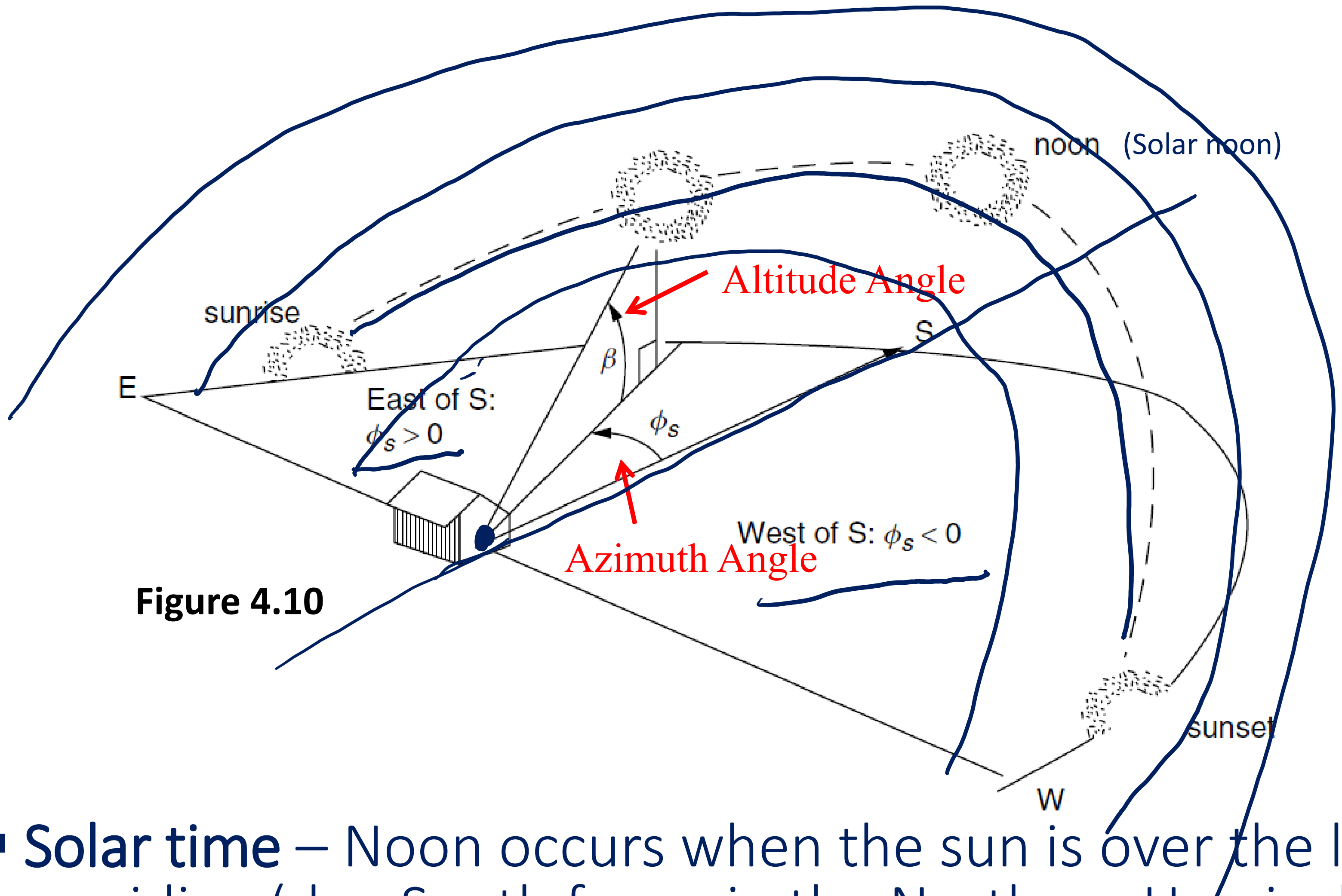
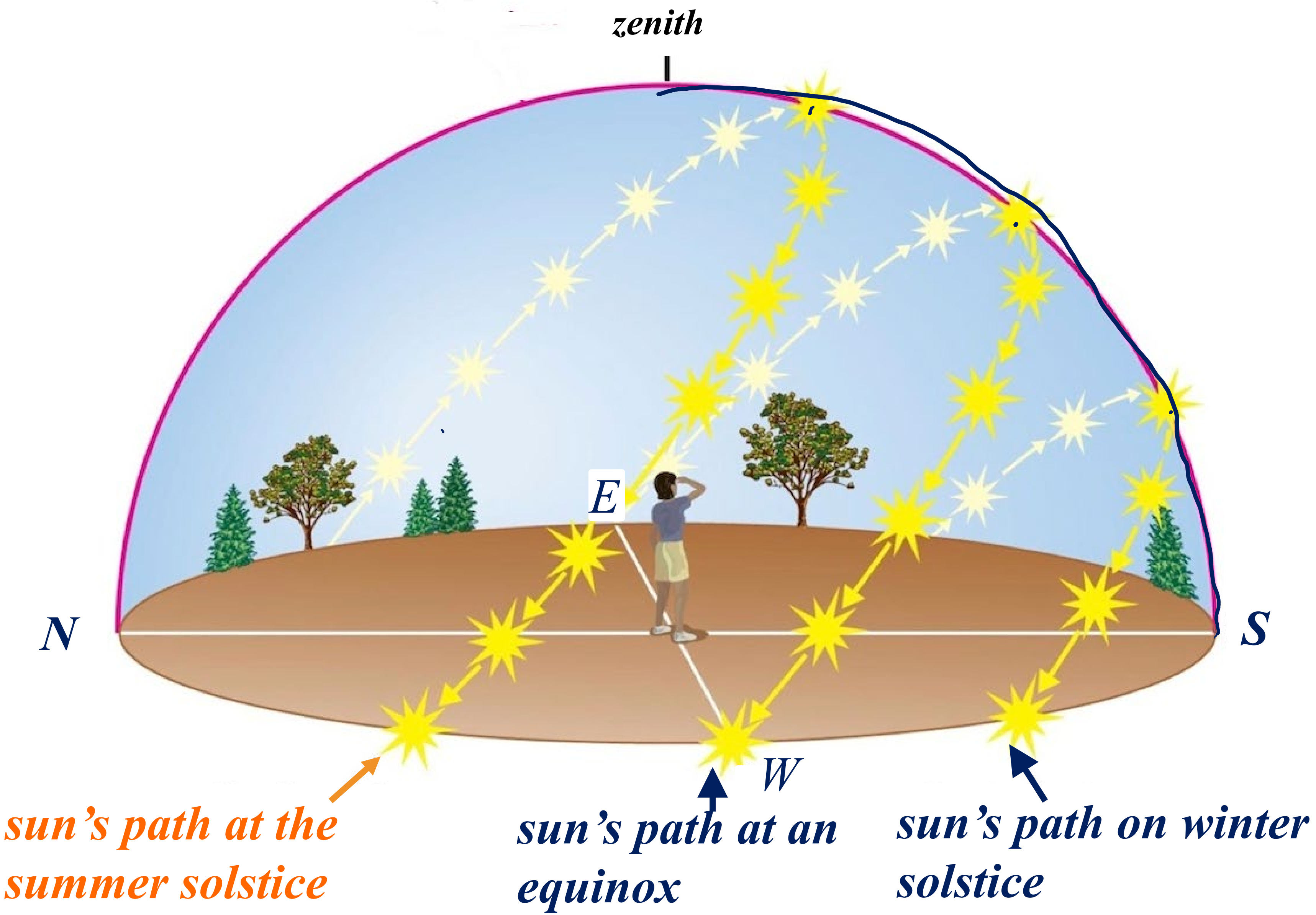


Figure 4.10

- **Solar time** – Noon occurs when the sun is over the local meridian (due South for us in the Northern Hemisphere above the tropics)

Sun Path


zenith
E
N
S
W
sun's path at the summer solstice
sun's path at an equinox
sun's path on winter solstice

Hour Angle

- **Hour angle** *H*- the number of degrees the earth must rotate before sun will be over the local line of longitude
- If we consider the earth to rotate at 15˚/hr, then

$$\text{Hour angle } H = \left(\frac{15^\circ}{\text{hour}}\right) \cdot (\text{hours before solar noon}) \qquad (7.10)$$

- Examples:

(4.10)

  - At 11 AM **solar time**, *H* = +15˚ (the earth needs to rotate 1 more hour to reach solar noon)
  - At 2 PM **solar time**, *H* = -30˚

Solar Coordinates

- Calculate the position of the Sun at any time of day on any day of the year:

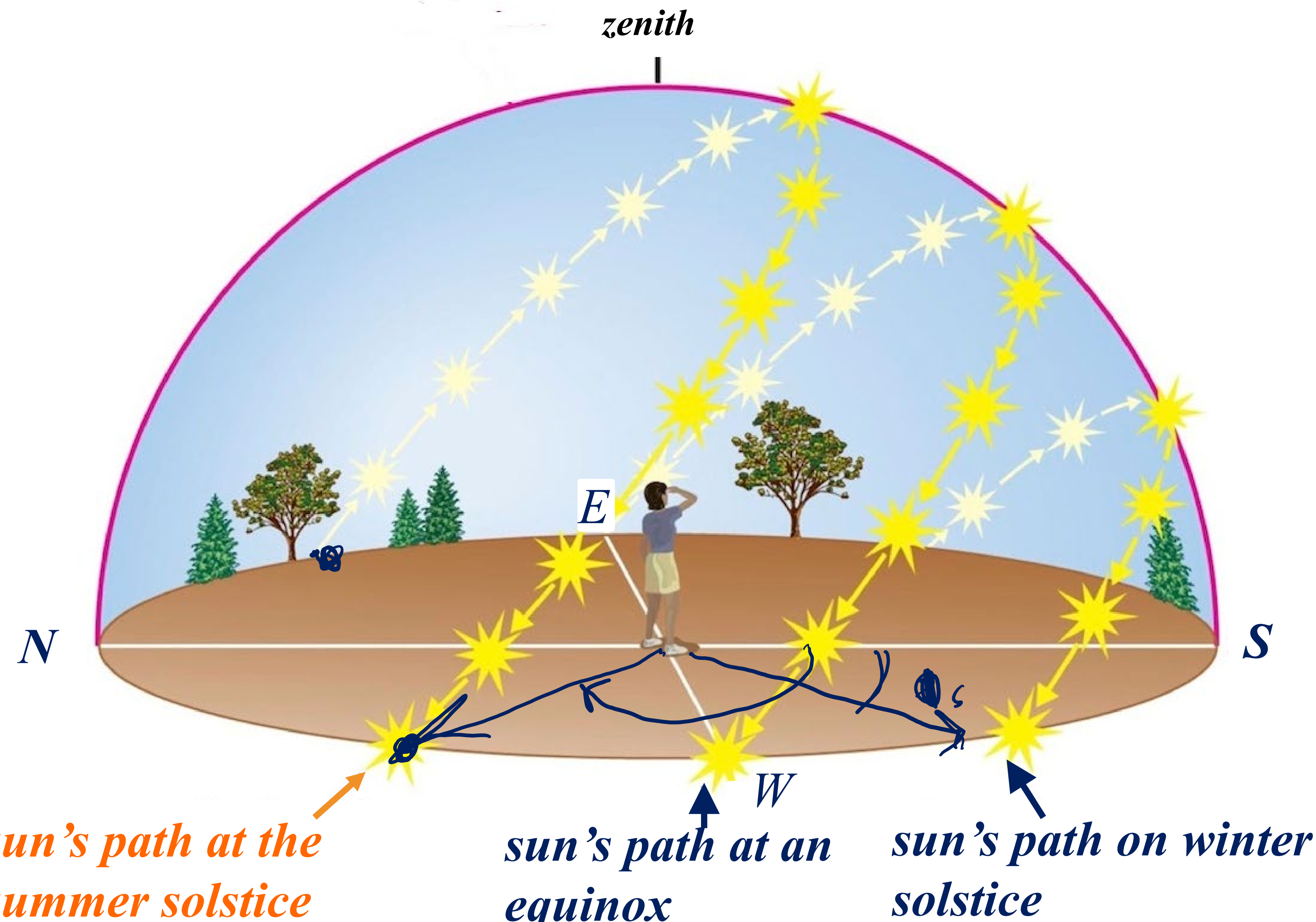
$$\sin \beta = \cos L \cos \delta \cos H + \sin L \sin \delta \tag{4.8}$$

$$\sin \phi_S = \frac{\cos \delta \sin H}{\cos \beta} \tag{4.9}$$

- **Be careful!** In Spring and Summer, the Sun can be more than 90° from due South at sunset/sunrise:

(4.11)

Sun Path



zenith
E
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sun's path at the summer solstice
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sun's path on winter solstice

Solar Coordinates

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$$\sin\beta = \cos L \cos\delta \cos H + \sin L \sin\delta \qquad (4.8)$$

$$\sin\phi_S = \frac{\cos\delta \sin H}{\cos\beta} \qquad (4.9)$$

- **Be careful!** In Spring and Summer, the Sun can be more than 90° from due South at sunset/sunrise:

$$\text{if} \quad \cos H \geq \frac{\tan\delta}{\tan L}, \qquad \text{then } |\phi_S| \leq 90^\circ; \qquad \text{otherwise } |\phi_S| > 90^\circ \qquad (4.11)$$

Example: Where is the Sun?

- Do Example 4.3, p.198 in text
  - Find altitude and azimuth of the sun at 3:00 pm solar time in Boulder, CO (L=40 degrees) on the Summer Solstice

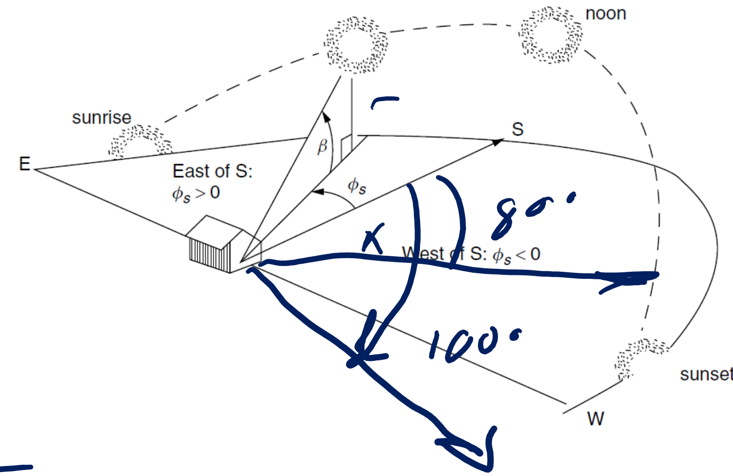
@ S.S, $\delta = 23.45^\circ$

$H = \frac{15^\circ}{h} \cdot (-3h) = -45^\circ = H$

$\sin\beta = \cos l \cdot \cos\delta \cdot \cos H + \sin l \cdot \sin\delta$

$\sin\beta = 0.7527$

$\beta = \sin^{-1}(0.7527) = 48.8^\circ$



Extra room for problem

azimuth angle – eq 4.9

$$\sin \phi_s = \frac{\cos\delta \cdot \sin H}{\cos \beta} = -0.9848$$

$\phi_s = \sin^{-1}(-0.9848) = -80°$ or $180° - (-80°) = 260°$

↓ 80° w of south

↓ 100° w of south

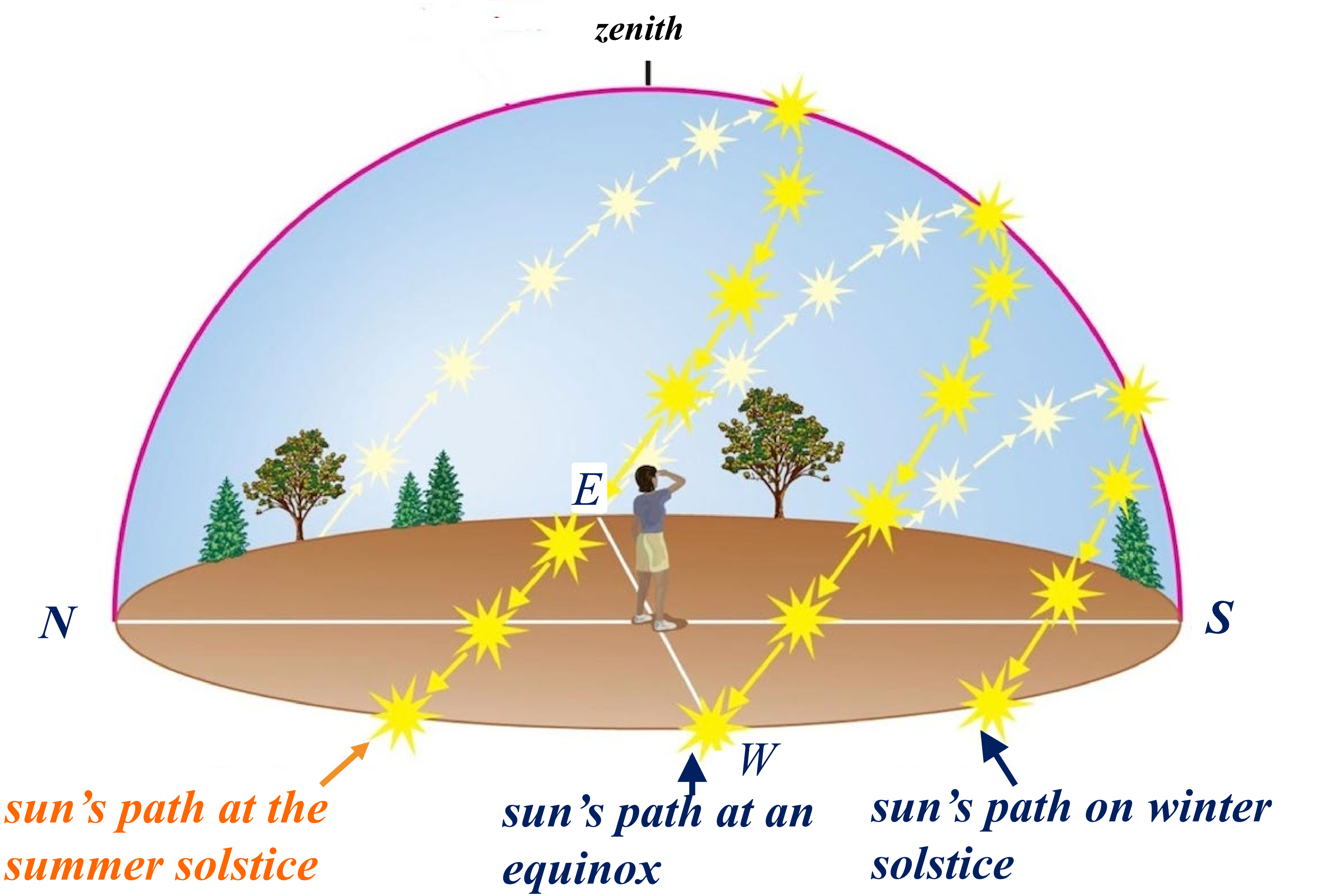
To find correct eq 4.11

$\cos H = \cos(-45°) = 0.707$ $\qquad \frac{\tan\delta}{\tan L} = 0.917$

$\cos H \geq \frac{\tan\delta}{\tan L} \Rightarrow \phi = -80°$ ✓

$|\phi_s| \leq 90°$

Sun Path



zenith
E
N
S
W
sun's path at the summer solstice
sun's path at an equinox
sun's path on winter solstice

Sun Path Diagram for Shading Analysis: 40°N Latitude

$$\beta_N = 90° - L + \delta \quad (4.7)$$

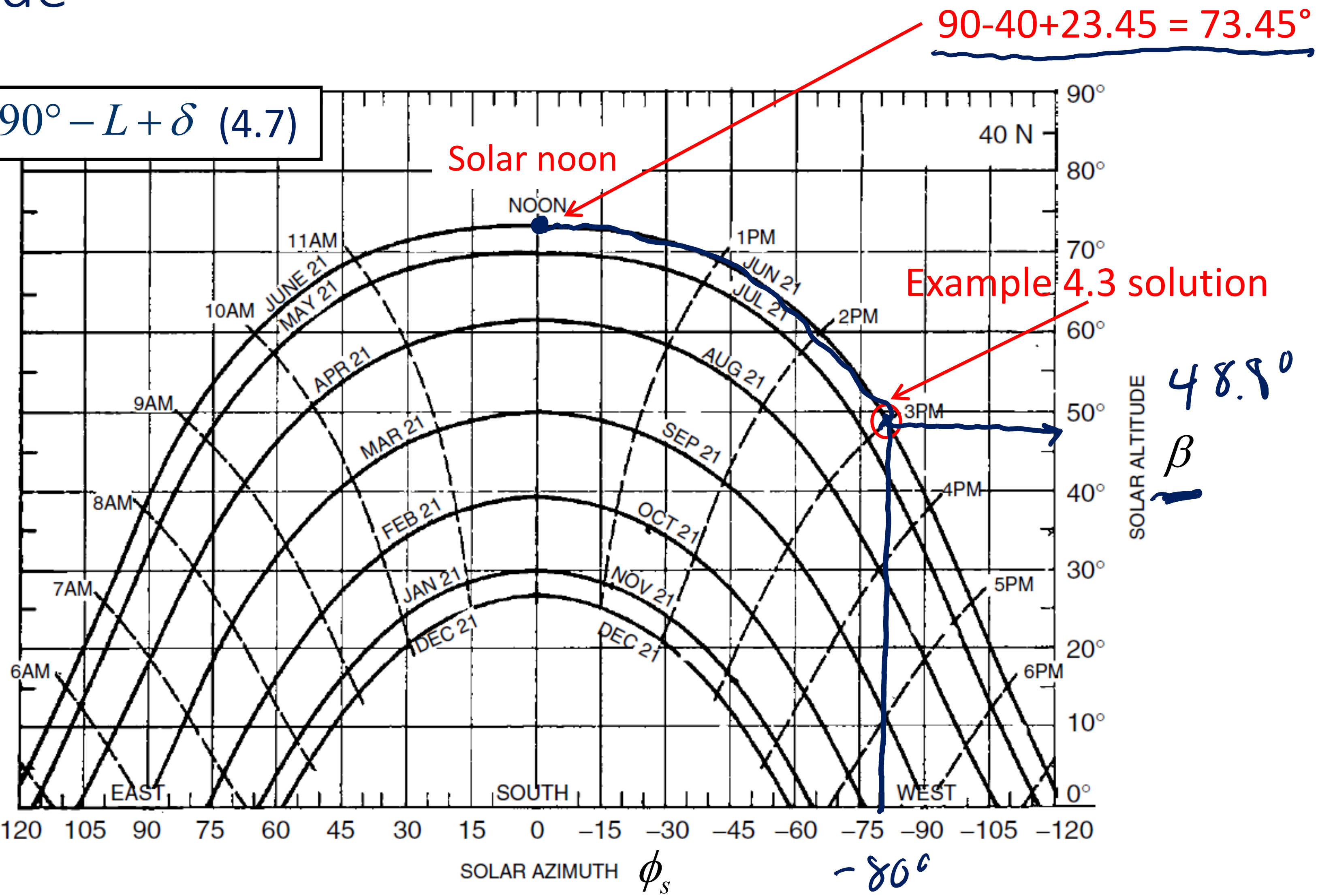
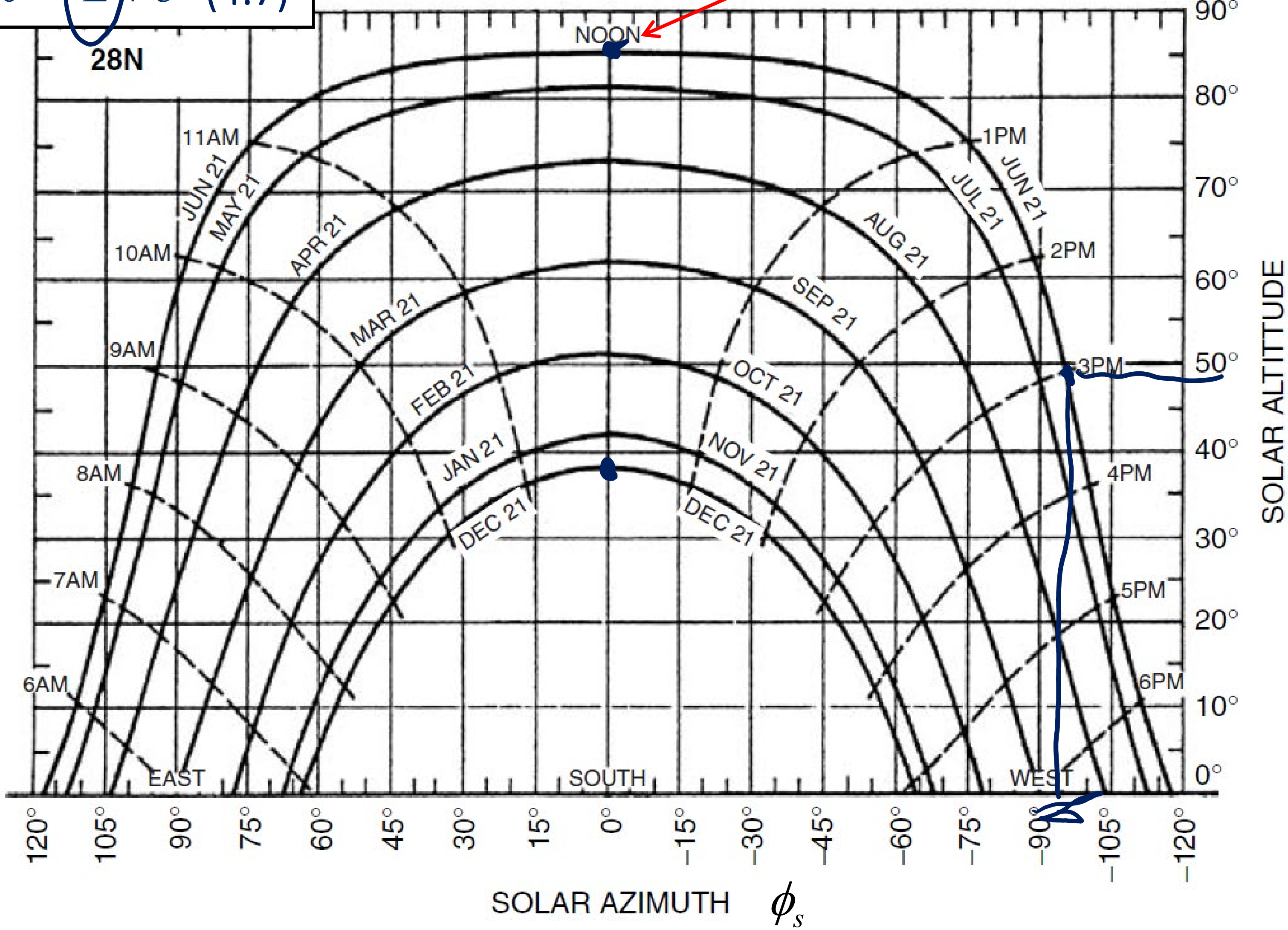


Figure 4.12 A sun path diagram showing solar altitude and azimuth angles for 40° latitude. Diagrams for other latitudes are in Appendix B.

Sun Path Diagram: 28°N Latitude

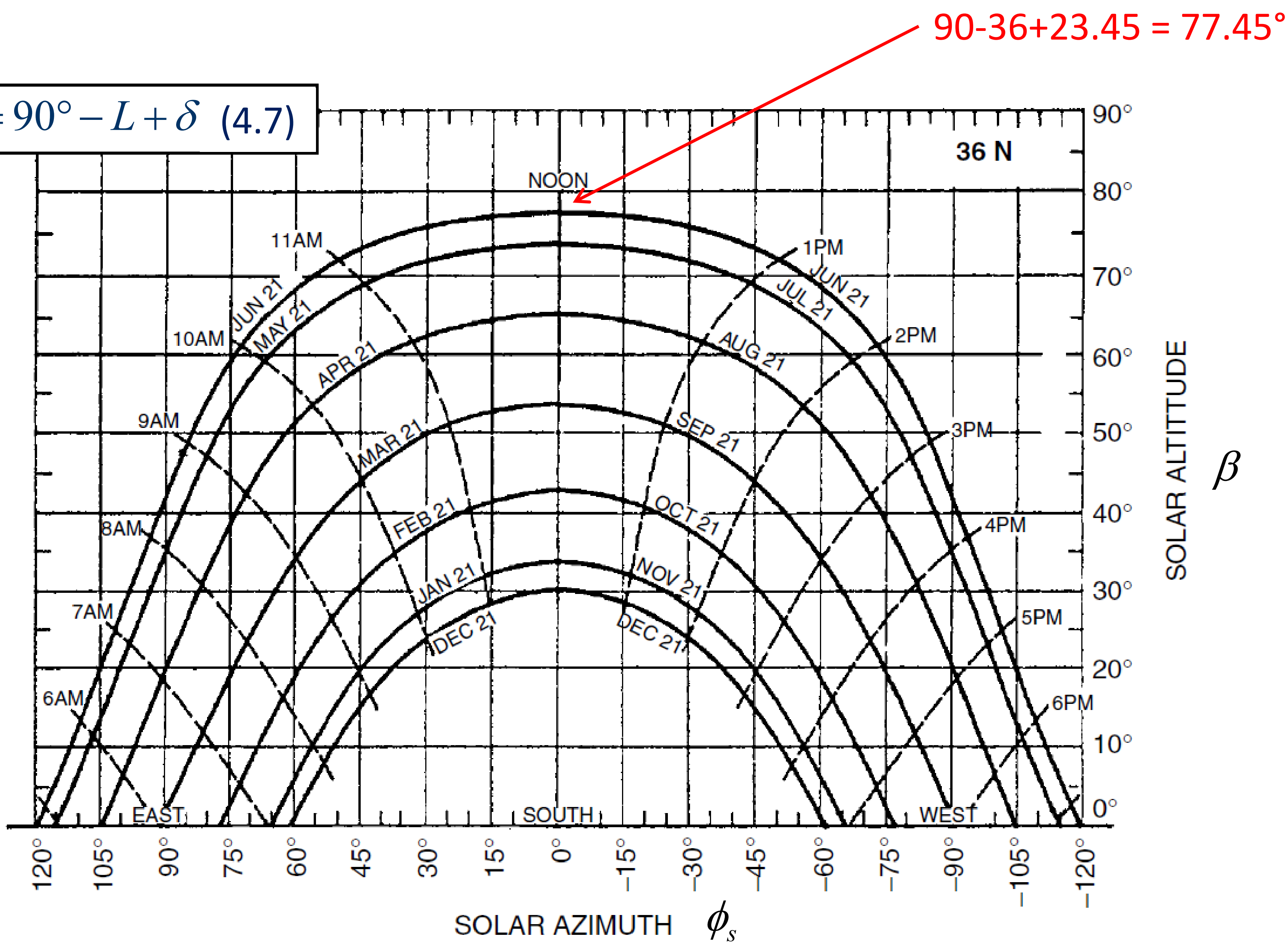
$$\beta_N = 90° - L + \delta \quad (4.7)$$

90-28+23.45 = 85.45°



Sun Path Diagram: 36°N Latitude

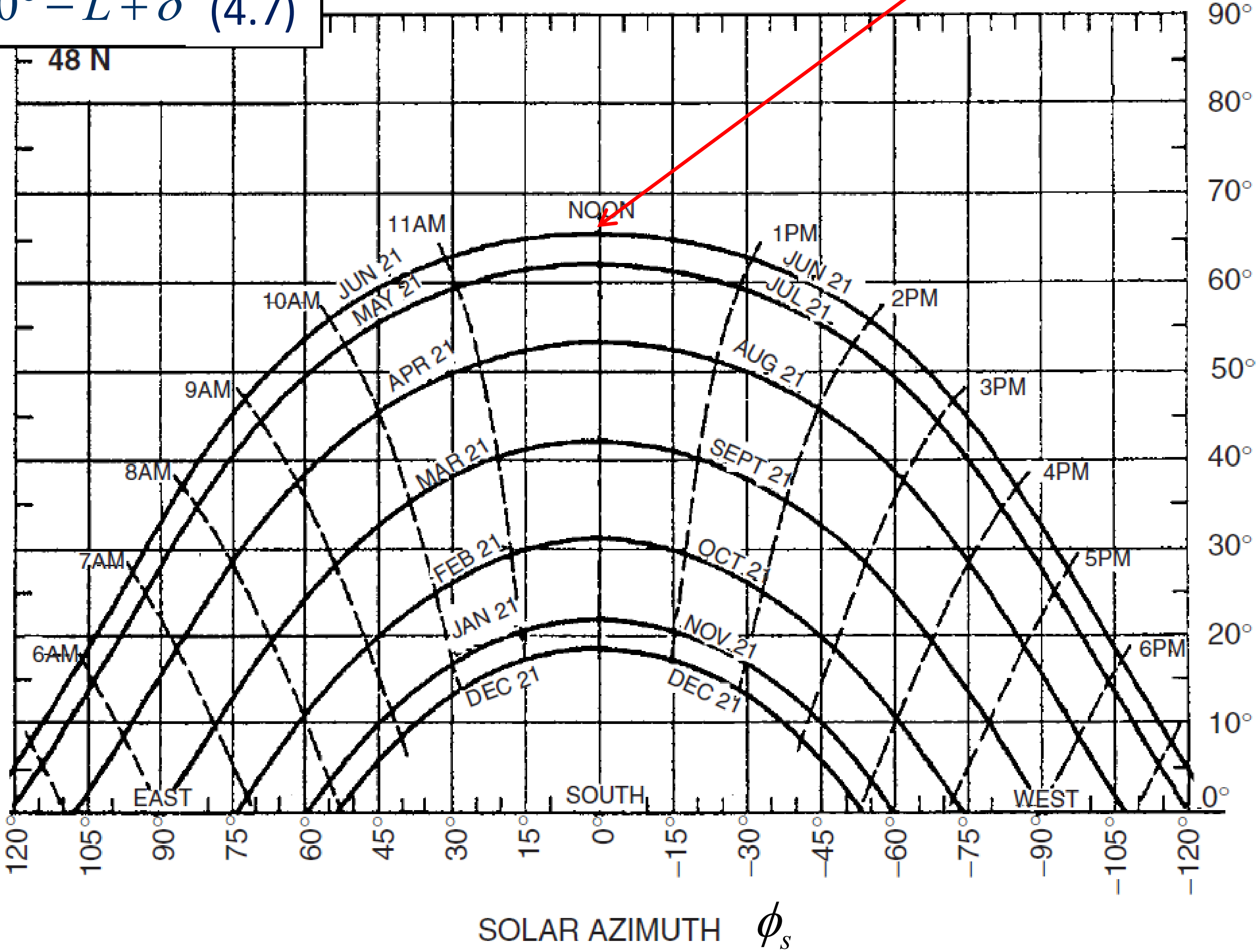
$$\beta_N = 90° - L + \delta \quad (4.7)$$



Sun Path Diagram: 48°N Latitude

$$\beta_N = 90° - L + \delta \quad (4.7)$$

90-48+23.45 = 65.45°



β

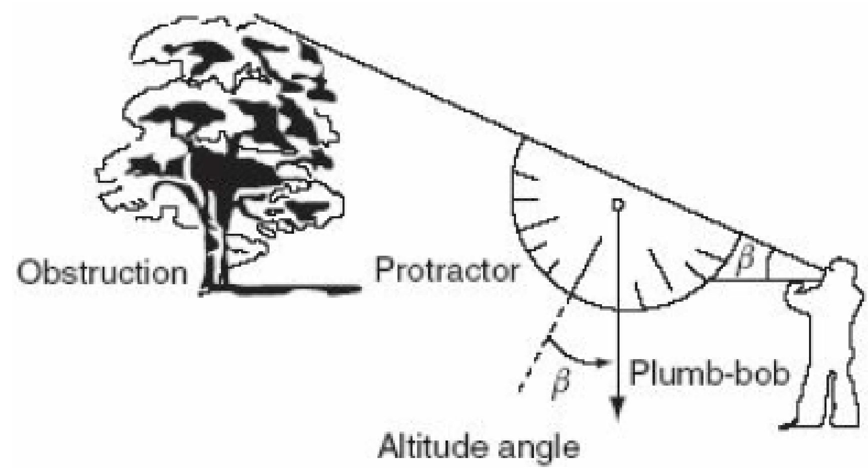
ϕ_s

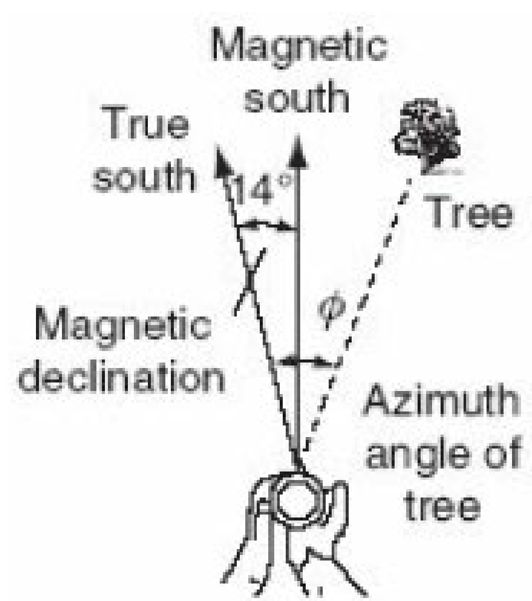
4.5: Sun Path Diagrams for Shading Analysis

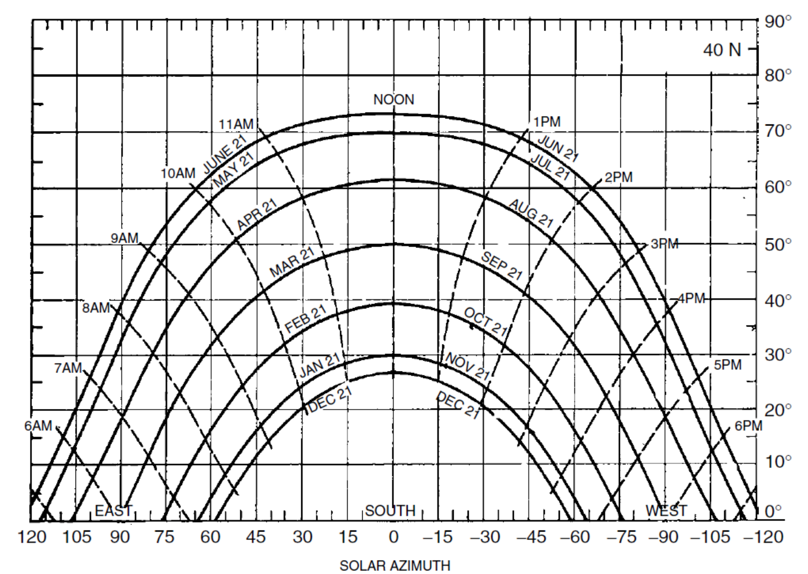
- Now we know how to locate the sun in the sky at any time
- This can also help determine what sites will be in the shade at any time
  - Sketch the azimuth and altitude angles of trees, buildings, and other obstructions
  - Sections of the sun path diagram that are covered indicate times when the site will be in the shade
- Shading of a portion of a solar panel could greatly reduce the output for the full panel (depending upon design)

Sun Path Diagram for Shading Analysis

- Use a simple plumb-bob, protractor and compass to put obstructions on the diagram

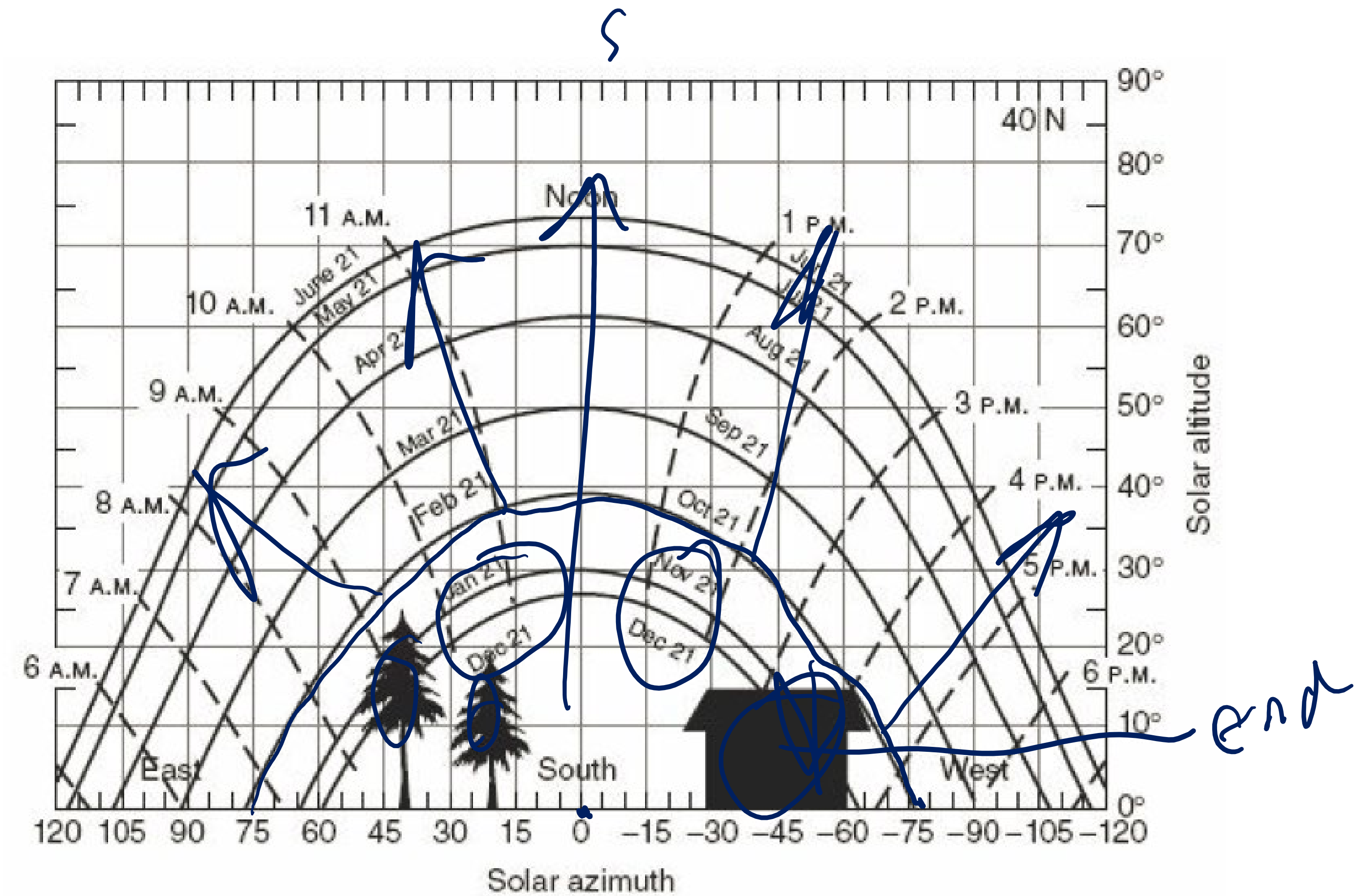






Sun Path Diagram for Shading Analysis

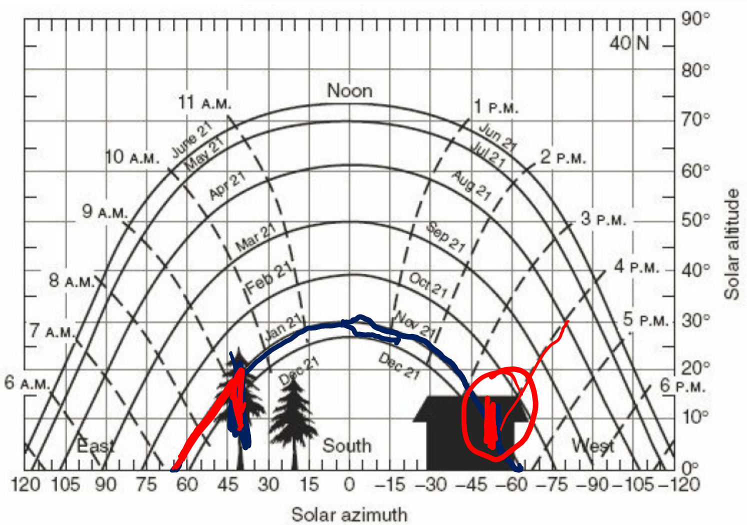
- Trees to the southeast, small building to the southwest
- Can estimate the amount of energy lost to shading



Sun Path Diagram for Shading Analysis

TABLE 4.2: Hour-by-Hour (W/m^2) and Daylong (kWh/m^2) Clear Sky Insolation at 40° Latitude in January for Tracking and Fixed, South-Facing Collectors

	Tracking		Fixed, South-Facing Tilt Angles						
Solar Time	One-axis	Two-axis	0	20	30	40	50	60	90
7, 5	0	0	0	0	0	0	0	0	0
8, 4	439	462	87	169	204	232	254	269	266
9, 3	744	784	260	424	489	540	575	593	544
10, 2	857	903	397	609	689	749	788	803	708
11, 1	905	954	485	722	811	876	915	927	801
12	919	968	515	761	852	919	958	968	832
$kWh/m^2/d$	6.81	7.17	2.97	4.61	5.24	5.71	6.02	6.15	5.47



Ex. 4.4: January day; south-facing collector; at 40° N latitude with a fixed, 30° tilt angle.

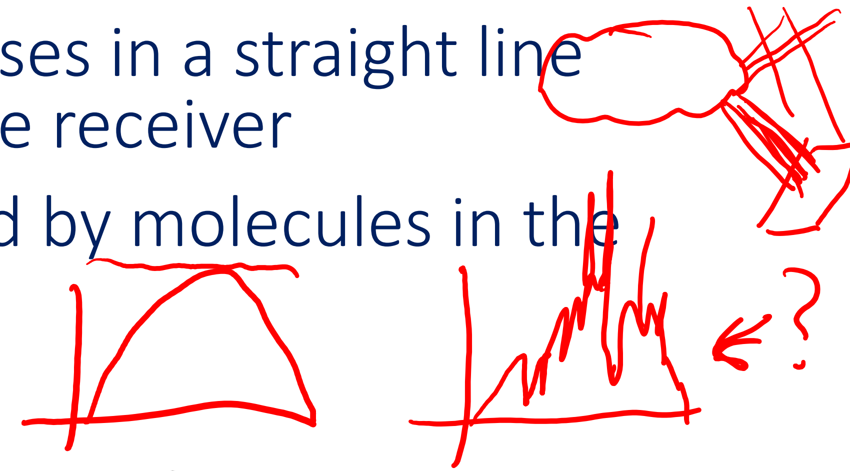
4.9: Clear-Sky Direct-Beam Radiation

How much sunlight reaches us?

- We now know where the Sun is at any given time at any location on Earth
- Based on this, how much solar insolation can we expect at a given site?
  - This will help us determine how much energy can be expected from a solar panel installation

Clear Sky Direct-Beam Radiation

- **Direct beam radiation** I_{BC} – passes in a straight line through the atmosphere to the receiver
- **Diffuse radiation** I_{DC} – scattered by molecules in the atmosphere



- **Reflected radiation** I_{RC} – bounced off a surface near the reflector

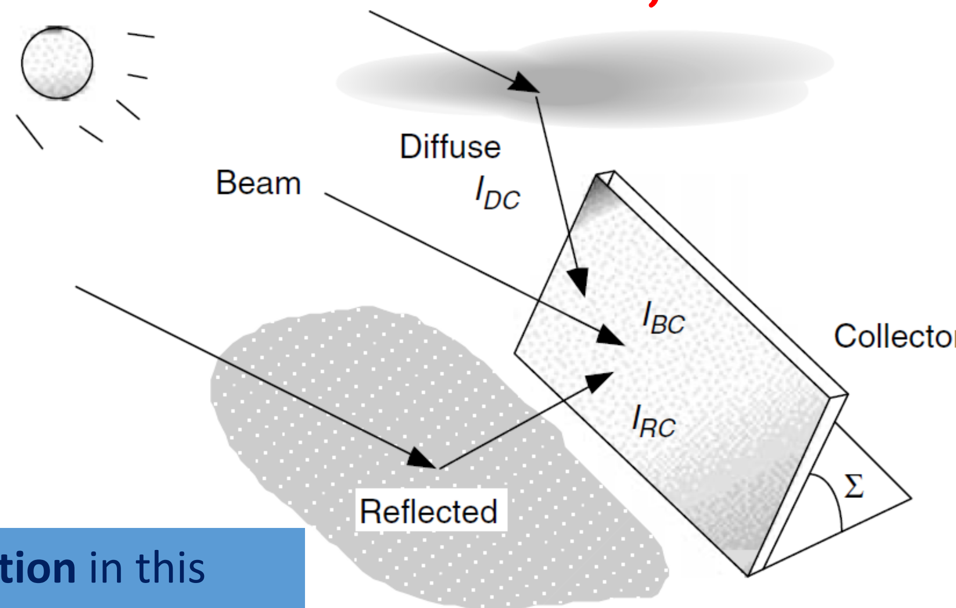


Figure 4.19

We'll only focus on **direct beam radiation** in this class.

Extraterrestrial Solar Insolation I_0

- I_0 is the starting point for clear sky radiation calculations
- I_0 passes perpendicularly through an imaginary surface outside of the earth's atmosphere

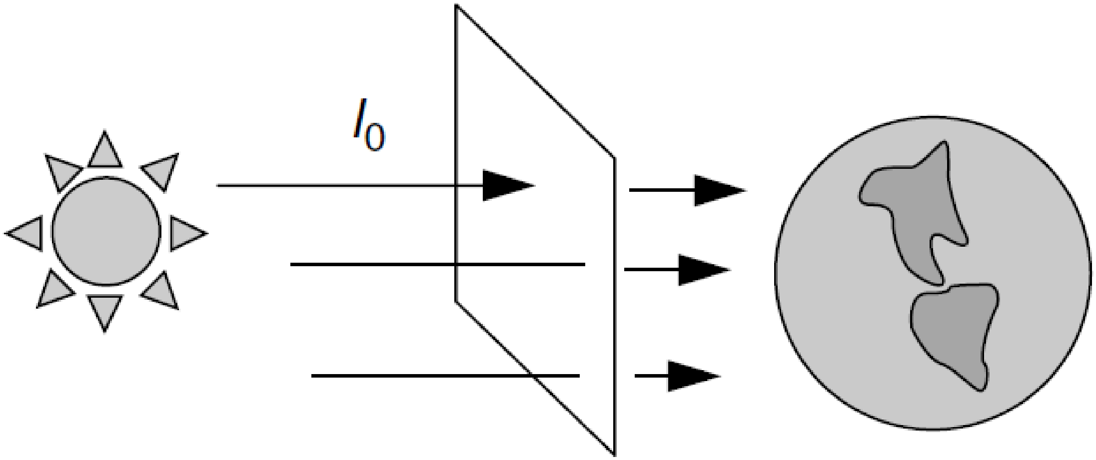
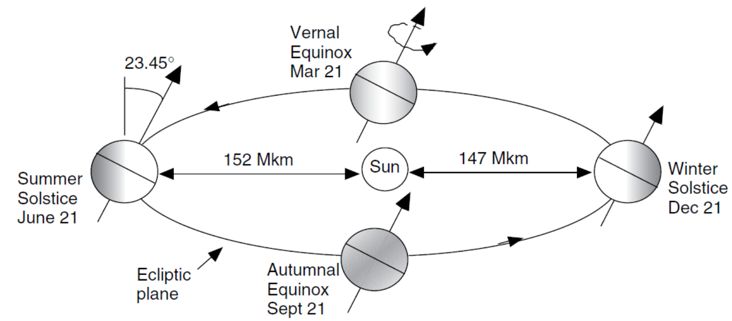


Figure 4.20

Extraterrestrial Solar Insolation I_0

- I_0 varies with the Earth's distance from the sun as well as sunspots and other solar activity
- We will **ignore sunspot effects**
- We can approximate I_0 as:



$$I_0 = SC \cdot \left[1 + 0.034 \cos\left(\frac{360n}{365} \right) \right] \quad (\mathrm{W/m^2}) \qquad (4.19)$$

SC = *solar constant* = 1.377 kW/m^2

n = day number

Extraterrestrial Solar Insolation I_0

- Much of I_0 is absorbed by various gases, scattered by dust, air molecules, water vapor, etc.
- In one year, less than half of I_0 reaches earth's surface as a direct beam
- On a sunny, clear day, beam radiation may exceed 70% of I_0

Attenuation of Incoming Radiation

$$I_B = Ae^{-km} \qquad (4.20)$$

- I_B = beam portion of the radiation that reaches the earth's surface
- A = apparent extraterrestrial flux
- k = optical depth
- m = air mass ratio

The *A* and *k* values are location dependent, varying with values such as **dust** and **water vapor** content

$\frac{H_2}{H_1}$

$$\text{Air mass ratio } m = \sqrt{(708 \sin\beta)^2 + 1417} - 708 \sin\beta \qquad (4.21)$$

Attenuation of Incoming Radiation

- *A* and *k* values can be found from **empirical** data:

TABLE 4.5 Optical Depth *k*, Apparent Extraterrestrial Flux *A* for the 21st Day of Each Month

Month:	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
A (W/m^2):	1230	1215	1186	1136	1104	1088	1085	1107	1151	1192	1221	1233
k:	0.142	0.144	0.156	0.180	0.196	0.205	0.207	0.201	0.177	0.160	0.149	0.142

Source: ASHRAE (1993).

*This table is based on empirical data for a moderately dusty atmosphere with atmospheric water vapor content equal to the average monthly values in the US.

Attenuation of Incoming Radiation

- *A* and *k* values can also be found from a best fit equation based on measured data:

$$A = 1160 + 75 \sin\left[\frac{360}{365}(n - 275)\right] \quad (\mathrm{W/m^2}) \qquad (4.22)$$

$$k = 0.174 + 0.035 \sin\left[\frac{360}{365}(n - 100)\right] \qquad (4.23)$$

*Best fit equations based on Table 4.5 data

Example 4.8: Direct Beam Radiation at Earth's Surface

- Find I_B (the direct beam solar radiation) at solar noon on a clear day in Atlanta ($L = 33.7°$) on May 21st. Compare empirical calculation (Table 4.5) to the best-fit equations (4.21) through (4.23)

$L = 33.7$, May 21 => $n = 141$

E 4.22 $A = 1160 + 75 \sin\left(\frac{360}{365}(n - 275)\right) = 1104\ w/m^2 = A$ (with $n = 141$)

$K = 0.174 + 0.035 \sin\left[\frac{360}{365}(n-100)\right] = 0.197 = K$

$\delta = 23.45 \sin\left(\frac{360}{365}(141 - 81)\right) = 20.14°$ (with $n = 141$)

$\beta_N = 90° - L + \delta = 90 - 33.7 + 20.1 = 76.4°$

Room for problem

$m = \sqrt{(708 \sin\beta)^2 + 1417} - 708 \sin\beta = 1.024 = m$

Eq 4.20

$I_\beta = A e^{-km} = 1104 e^{-0.197 \times 1.024} = 902 \text{ W/m}^2$

Wrap-up

- That’s it for Chapter 4
- No HW this week – start studying for Exam 2
- I’m now a barber and part time shop teacher