Today

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Review
More Economics

Review

Time value of money \$1.00 now $\neq$ \$1.00 in a year

F = Future value \$

P = Present value \$

A = annual value \$/y

d = discount rate (interest for example)

$\beta^n = (1+d)^{-n}$ = single payment present worth factor

$$P = F(1+d)^{-n} = F \cdot \beta^n$$

Cash Flows

+ = inflow = I get money

− = outflow = I pay money

Net present value => move all cash flows to the present to compare apples to apples

Annual costs

$P = A \cdot PVF(d,n)$ $\quad$ $PVF(d,n)$ = present value function $= \dfrac{(1+d)^n - 1}{d(1+d)^n} = \dfrac{1-\beta^n}{d}$

$A = P \cdot CRF(d,n)$ $\quad$ $CRF(d,n)$ = capital recovery factor $= \dfrac{d(1+d)^n}{(1+d)^n - 1} = \dfrac{d}{1-\beta^n}$

$$CRF(d,n) = \frac{1}{PVF(d,n)}$$

Discount rate Implications:

Consider: a project will return a savings of \$10,000 at the end of 5 years

$P = F\beta^n = 10,000(1+d)^{-5}$

For $d = 0.1$ => $P = \$6,201$

For $d = 0.2$ => $P = \$4,019$

In general, for a given future worth, the lower the discount rate, the higher the present worth

Another way to state this: the lower the discount rate, the more valuable a future payoff becomes.

Life Cycle Costs Example

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Lets consider two 100 hp motors - a and b to be used over a 20-year period. The discount rate is given at 10%

specs:

motor	cost	load
a	\$2,400	79 kW
b	\$2,900	77.5 kW

- motors are used 1,600 hours/year
- cost of electricity is 0.08 \$/kWh → assumed constant

which is the better investment? <=> which has the lowest life-cycle cost?

Let's look at yearly energy cost:

$A^a = 79\text{ kW} \times 1600\text{ h} \times 0.08\,\$/\text{kWh} = \$10{,}112$ → will pay this each year

$A^b = 77.5\text{ kW} \times (\text{same thing}) = \$9{,}920$

$P^a = 2400 + A^a \cdot PVF(d,n) = 2400 + A^a \dfrac{1-\beta^n}{d} = 2400 + 10{,}112 \underbrace{\dfrac{(1-1.1^{-20})}{0.1}}_{8.5136} = \$88{,}484$

$P^b = 2900 + A^B \cdot PVF(d,h) = 2900 + \$9{,}920 \times 8.5136 = \$87{,}354$

$P^a - P_b = \$1{,}135 \Rightarrow$ motor b purchase results in a \$1,135 savings despite the higher purchase price.

Infinite Horizon Cash Flow Sets

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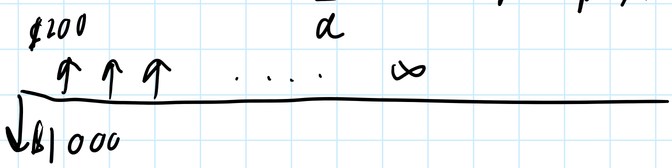
Consider an infinite, uniform cash-flow set
↳ examples: Invest money, withdraw small part for rest of life

as $n \to \infty$ $\quad \to \infty \qquad \to 0$ as $n \to \infty$

$$P = A\,\frac{(1-\beta^n)}{\alpha} = A\left(\frac{1-(1+\alpha)^{-n}}{\alpha}\right) = A\,\frac{1}{\alpha}$$

$A = \alpha \cdot P \to$ this makes sense. For an Investment P earning interest α, I can withdraw the yearly earnings ($\alpha \times P$) for ever.

In this case, $\alpha =$ "simple rate of return"
$\frac{1}{\alpha}$ = simple payback



$P = \$1000$
$A = \$200$
$\alpha = \frac{A}{P} = 0.2 = 20\%$

simple payback period = 5 years

$5 \times 200 = 1000$ ✓

Internal Rate of Return

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So far we've been given the interest/discount rate.
what if we need to determine the discount rate at which it makes sense to do the project?

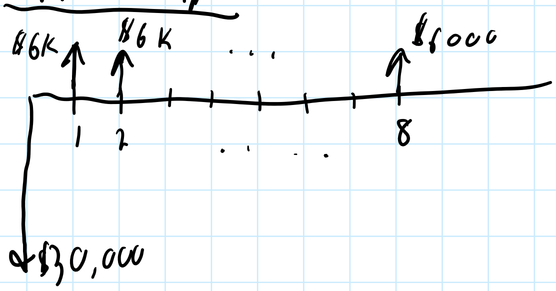
→ This is called the Internal Rate of Return (IRR)
  ↳ Higher is better: "This project makes sense up to a d of ___"

→ IRR = Break even discount rate = speed at which the returns recover an investment

$$P - \sum_{t=1}^{n} A_t \beta^t = 0 \iff \text{Solve for } d \text{ when } NPV = 0$$

- Note that there is not a closed form solution
  ↳ look up in table, use Excel or TI-86

IRR Example



IRR: Solve for the d that makes NPV = 0

$$P = -30{,}000 + 6000 \cdot \frac{1-(1+d)^{8}}{d} = 0$$

$$PVF(d,8) = \frac{30{,}000}{6{,}000} = 5$$

Internal Rate of Return (%)

Life (yr)	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	16%	18%	20%	22%
5	4.58	4.45	4.33	4.21	4.10	3.99	3.89	3.79	3.60	3.43	3.27	3.13	2.99	2.86
6	5.42	5.24	5.08	4.92	4.77	4.62	4.49	4.36	4.11	3.89	3.68	3.50	3.33	3.17
7	6.23	6.00	5.79	5.58	5.39	5.21	5.03	4.87	4.56	4.29	4.04	3.81	3.60	3.42
8	7.20	6.73	6.46	6.21	5.97	5.75	5.53	5.33	4.97	4.64	4.34	4.08	3.84	3.62
9	7.79	7.44	7.11	6.80	6.52	6.25	6.00	5.76	5.33	4.95	4.61	4.30	4.03	3.79
10	8.53	8.11	7.72	7.36	7.02	6.71	6.42	6.14	5.65	5.22	4.83	4.49	4.19	3.92
11	9.25	8.76	8.31	7.89	7.50	7.14	6.81	6.50	5.94	5.45	5.03	4.66	4.33	4.04
12	9.95	9.39	8.86	8.38	7.94	7.54	7.16	6.81	6.19	5.66	5.20	4.79	4.44	4.13
13	10.63	9.99	9.39	8.85	8.36	7.90	7.49	7.10	6.42	5.84	5.34	4.91	4.53	4.20
14	11.30	10.56	9.90	9.29	8.75	8.24	7.79	7.37	6.63	6.00	5.47	5.01	4.61	4.26
15	11.94	11.12	10.38	9.71	9.11	8.56	8.06	7.61	6.81	6.14	5.58	5.09	4.68	4.32
20	14.88	13.59	12.46	11.47	10.59	9.82	9.13	8.51	7.47	6.62	5.93	5.35	4.87	4.46
25	17.41	15.62	14.09	12.78	11.65	10.67	9.82	9.08	7.84	6.87	6.10	5.47	4.95	4.51
30	19.60	17.29	15.37	13.76	12.41	11.26	10.27	9.43	8.06	7.00	6.18	5.52	4.98	4.53

[a]Enter the row with project life, move across to values close to your simple payback, move up to read IRR. A 15-year project with a 5-year simple payback has an IRR a bit over 18%.

Inflation

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General increase in level of prices in an economy. No single index of it, depends on the product. What you want to buy today is different than what you'd buy in 1972. want inflation rate small and positive

- ↳ can view inflation as a decline in the value of the purchasing power of money
- ↳ Consumer price index (CPI) uses a basket of goods and services as a proxy
  - ↳ reference basis is year 1967

what if we have a 5% interest rate and 5% inflation?

ans: they cancel out $1.00 now = $1.00 next year in terms of purchasing power

Equivalent discount rate

d' = Equivalent discount rate
j = Inflation

$$\frac{1+j}{1+d} = \frac{1}{1+d'} \Rightarrow d' = \frac{d-j}{1+j} \quad \text{and} \quad j = \frac{d-d'}{1+d'}$$

Let's test this out:

$$j = 5\%,\ d = 5\% \Rightarrow d' = \frac{0}{1.05} = 0 \checkmark$$

$$j = 0,\ d = 5\% \Rightarrow d' = \frac{0.05}{1} = 0.05 \checkmark$$

$$j = 5\%,\ d = 0 \Rightarrow d' = \frac{-0.05}{1.05} \approx -0.05 \checkmark$$

Have to be careful about whether cashflow sets include inflation

indexed or constant worth cash flow does not take inflation into account

current or inflated or after inflation does

how do we use inflation?

1. Find d'
2. use d' instead of d
3. that is all

Example:

Motor selection example, but consider $j = 5\%$ (cost of electricity rises)

Find NPV using $j = 0.05$ and $d = 10\%$

$$d' = \frac{d-j}{1+j} = \frac{0.10-0.05}{1.05} = 0.0476$$

$1+j)$ $\quad 1.04$

$$P^a = 2400 + 10{,}112 \underbrace{\left(\frac{1 - 1.0476^{-20}}{0.0476}\right)}_{= 12.72} = \$131{,}022$$

$$P^b = 2400 + 9{,}920 \cdot 12.72 = \$129{,}082$$

$P^a - P^b = \$1940$ savings are larger than \$1,135 without electricity price escalation

Working at Home

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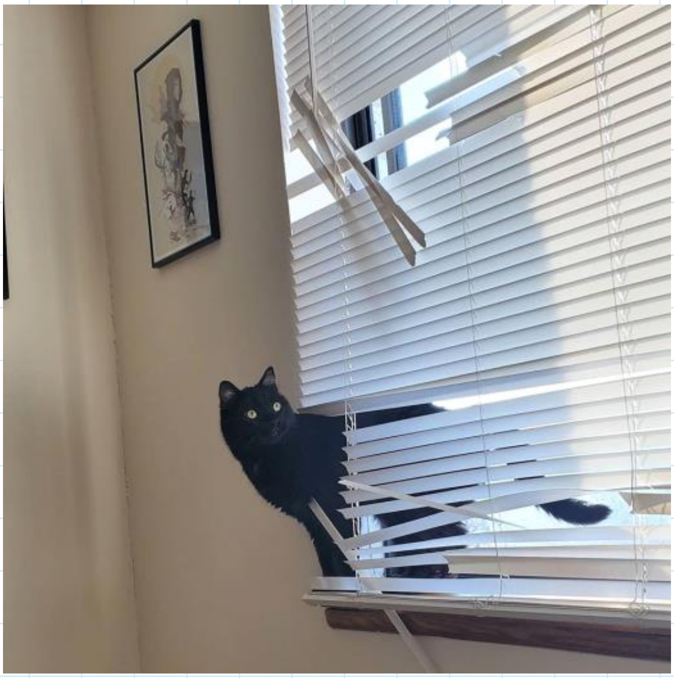
"Birds flying by? The blinds must die."



Catch up on reading: Hoopla, Libby apps
Catch up on Movies: The Current War!!!