Economics

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Motivation
Time value of money
Cash Flows
Equivalence
Inflation

Motivation

- As an Engineer, need to quantify costs.
  - ↳ Convince your boss that a project makes sense.
  - ↳ If it doesn't make sense economically, it won't happen.
  - ↳ Especially important for renewables. → Less experience with technology
  - ↳ Economics of renewables have become a large driver of adoption

Time value of money

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A dollar today is not the same as a dollar in one year.

- Would you rather have \$10 today or \$50 in 5 years?
- If you plan to make a \$50,000 purchase in 10 years, how much should you save today?

We'll learn how to answer these types of questions.

Let's start with a few terms:

F = future value (\$)

P = present value (\$)

A = annual value (\$/yr)

Start with something familiar: principle & interest

P => Principle: initial sum

i => interest: a measurement of the productivity of money over time - money today vs. money tomorrow

simple interest vs. compound interest - compound interest (what we consider) is when interest is also paid on the interest (vs. principle only). This difference is more notable when interest rate is higher.

Positive Interest Rate: This means that having \$1.00 in 10 years is not as good as having one dollar today because the assumption is that over 10 years you could do something better with that \$1.00 - use it to make more money.

↳ bank to earn interest (OK)

↳ invest (better)

so, for $i \geq 0$ Future Value > Present value

$F > P$

High level: \$1.00 is worth ≈ \$1.05 next year

$P < F$

Compound interest

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The convention is that payments always occur at the <u>end</u> of a period (e.o.p.).

e.o.p.	amount owed ($)	
0	P	← you borrow \$P at the end of year 0
1	$P(1+i)$	← then you owe \$P + \$(P×i) at the end of year 1
2	$P(1+i)^2$	← if another year goes by, you owe $P(1+i) + P(1+i)\times i$ ...
3	$P(1+i)^3$	
⋮	⋮	
n	$P(1+i)^n$	← call this F

Terms: $(1+i)^n$ "single payment compound factor"

$\beta^n = (1+i)^{-n}$ "single payment present worth factor"

$\beta \triangleq (1+i)^{-1}$

From the table: $F = P(1+i)^n$

or $P = F(1+i)^{-n} = F\beta^n$

Cash Flows

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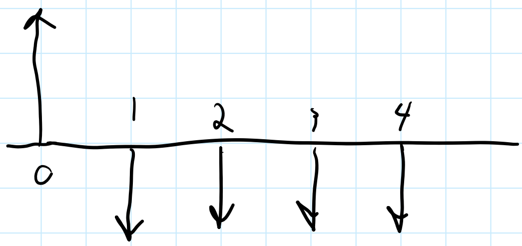
- It is useful to represent the time value of money concepts on a "cash flow diagram."
- A cash flow is a transfer of an amount A_t from one entity to another at time t.
- Convention: $+$ = inflow → I get money (this is still e.o.p.)
  $-$ = outflow → I pay money

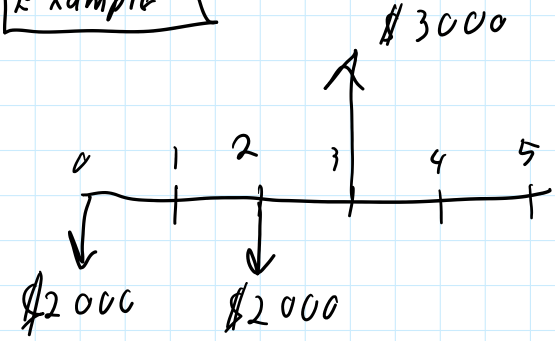


Ex I take out a loan at end of year 0 (present). Then I make 4 equal repayments over 4 years.

Inflows: revenue collected, loan recieved
Outflows: purchase made, payments made

- Each cash flow has (1) amount, (2) time, (3) sign.

Example



Let's say you have a project with these cash flows. How can you use this diagram to determine if this is a good project?

Zoom Quiz?

- First, what does this look like, just by inspection?
- To answer the first question, need to define the notion of equivalence for cash flows.

Equivalence (cash flows)

It is difficult to tell if a project makes sense from the cash flow diagram. This is because the payments are all in different years & the value of money in different years is not equivalent.

We saw that $F = P(1+i)^n$. This means that with an interest rate of i, \$P today is equivalent to \$F at the end of year n.

- Using this notion of equivalence, we can always take some amount K_j and "move" it to a future or past year.

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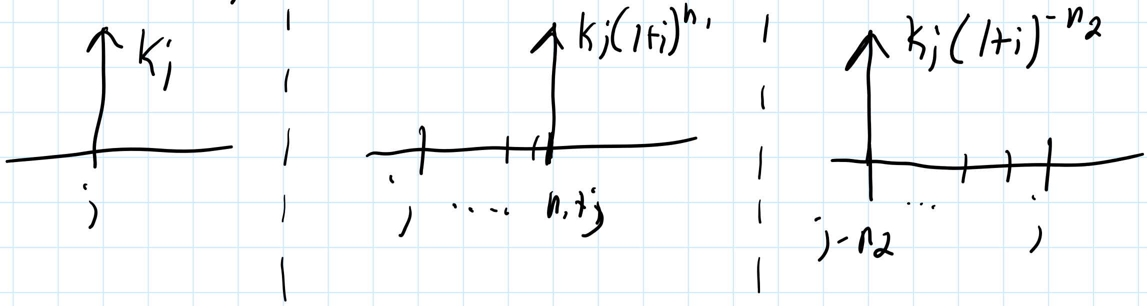
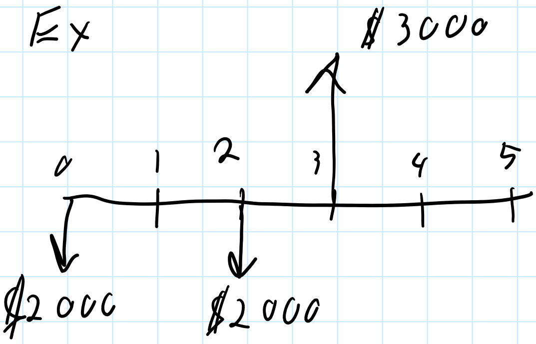
① move forward n_1 years → $K_j(1+i)^{n_1}$
② move backwards n_2 years → $K_j(1+i)^{-n_2}$

So the following 3 cash flows are equivalent



We can use equivalence to compare all cash flows at one point in time → this is the only way to compare "apples to apples"
↳ If 2 cash flows are equal at one point in time they are equal at all points in time.

Ex



$d = 12\%$ "discount rate"

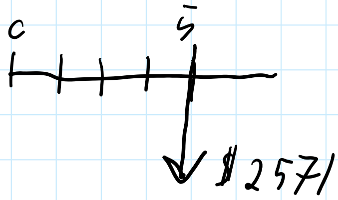
because used to "discount" cash flows to the present.
~ signifies the rate of earnings if put into the best possible investment.

a) what is the future worth of this cash flow set in year 5?

$$\begin{aligned} &-2000(1.12)^5 \\ &-2000(1.12)^3 \\ &+3000(1.12)^2 \\ \hline &-2571 \end{aligned}$$

⇒

$F = P(1+i)^n$

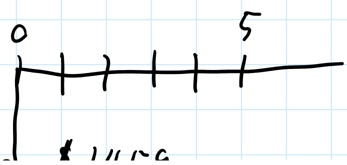


So the equivalent in year 5 is negative → not good

b) Present worth of the cash flow set in the present?

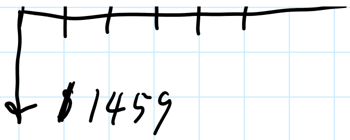
$$\begin{aligned} &-2000 \\ &-2000(1.12)^{-2} \\ &\underline{3000(1.12)^{-3}} \end{aligned}$$

⇒



$$-2000(1.12)^{-2}$$
$$3000(1.12)^{-3}$$
$$\overline{\quad -1459 \quad}$$

$\Rightarrow$



So these cash flows amount to -$1459 now or -$2571 in 5 years

check that these are equivalent:

$$F = P(1+i)^n$$
$$-\$1459(1.12)^5 = -\$2571 \checkmark$$

yes, for $d = 12\%$

Net present value

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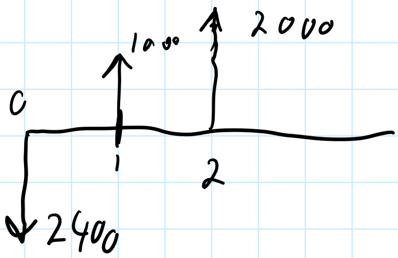
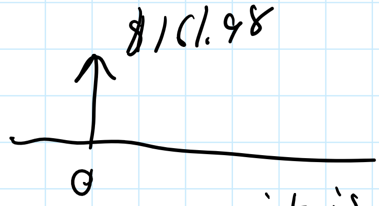
Moving all cash flows to the present is a good way to compare cash flow sets of different projects.
↳ This is called the "Net Present Value" (NPV)

Net Present Value Example

Suppose we have a project that returns
$1000 @ eoy 1 $i = 10\%$ (use this for d)
$2000 @ eoy 2
Also, suppose an initial investment of $2400 is required now.
What is the net present value?
Step 1: Draw the cash flows



=>



it is > 0 - good

$$P = -2400 + 1000(1.1)^{-1} + 2000(1.1)^{-2} = \$\,161.98$$

Net present value

- A standard approach to comparing cash flow sets is to compute the NPV for each project and compare them.
  Note: if the discount rate were lower, the NPV above would be higher

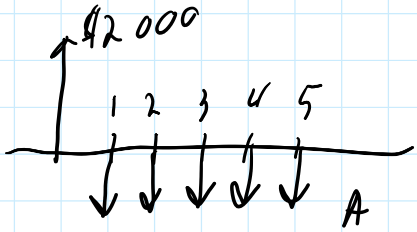
→ This leads to "Life cycle costs" - the present worth of a set of costs

- Now we can use the discount rate to "move" cash flows to different years for comparison purposes.
- Also want to be able to compute the equivalent <u>annual</u> cost for a set of cash flows.

<u>Example:</u> A capital investment project such as a renewable energy project requires funds - may be borrowed from a bank or investors or from owners.

An investment can be viewed as a loan w/ interest rate i that we want to pay back using a series of equal annual payments to pay back the loan with interest. → \$/yr

Again, we'll use this idea of moving cash flows to find A.



$d = 6\%$

Analysis is simplified if all cash flows have same value → uniform cash flows

what should A be? → find A such that the NPV is zero.

<u>Solution:</u> write down the equation for the NPV and then solve for A:

$$2000 - A(1.1)^{-1} - A(1.1)^{-2} \dots - A(1.1)^{-5} = 0$$

$$\underbrace{2000}_{P} = A\underbrace{\left[(1.1)^{-1} + (1.1)^{-2} + \dots (1.1)^{-5}\right]}$$

$$\rightarrow \sum_{t=1}^{n} \beta^{t} = \frac{(1+d)^n - 1}{d(1+d)^n}$$

$$\Rightarrow A = \frac{2000}{PVF(d,n)} = 2000\underbrace{\left[\frac{d(1+d)^n}{(1+d)^n - 1}\right]}_{\text{"Capital Recovery Factor" } CRF(d,n)}$$

⇒ from book, Appendix A.3
Present value function ⇒ $PVF(d,n)$
$P = A \cdot PVF(d,n)$

$$A = P \times CRF(d,n)$$

$A = \$474.79$ → So a \$2000 loan w/ interest rate of 6% can be paid back in 5 years with these annual payments