Lecture 12

3/12

Last time

Rotor Efficiency & Power Curves

Weibull & Rayleigh Wind distributions

Today

Logistics

Estimating Wind turbine Energy Production

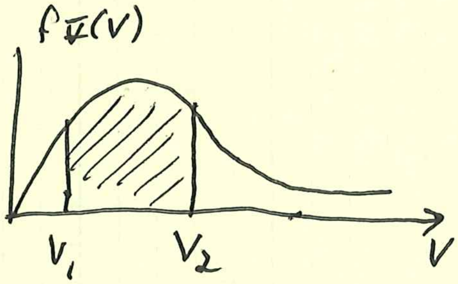
Recall: $f_{\underline{V}}(v)$ is a pdf

$f_{\underline{V}}(v) \geq 0 \;\forall V$

$\int_{-\infty}^{\infty} f_{\underline{V}}(v)\,dv = 1$

Prob that $V_1 < \underline{V} < V_2$

$= \int_{V_1}^{V_2} f_{\underline{V}}(v)\,dv$



For wind, pdf is weibull dist with $K=2$ (which makes it a Rayleigh dist) and we found that $\langle V\rangle = \frac{\sqrt{\pi}}{2} C$

$$\Rightarrow f_{\underline{V}}(v) = \frac{\pi v}{2\langle V\rangle^2} e^{\left[-\frac{\pi}{4}\left(\frac{v}{\langle V\rangle}\right)^2\right]}$$

we then showed

That $\langle P_w\rangle = \frac{1}{2}\rho Ar \langle V\rangle^3 \cdot \frac{6}{\pi}$

Note: this is power in the wind, before things like Betz's Limit is applicable

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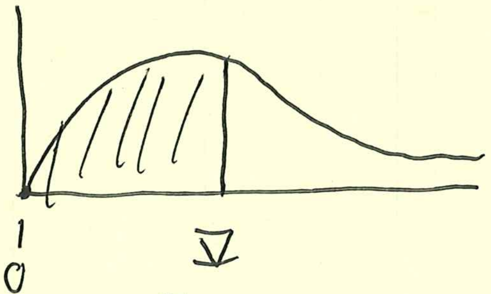
How can we use this?

One more tool: cumulative distribution function

Prob. that $v \le V = F(V) = \int_0^V f(v)dv$

=> nothing too special

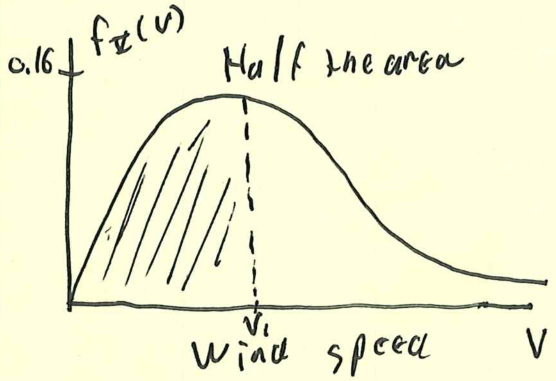


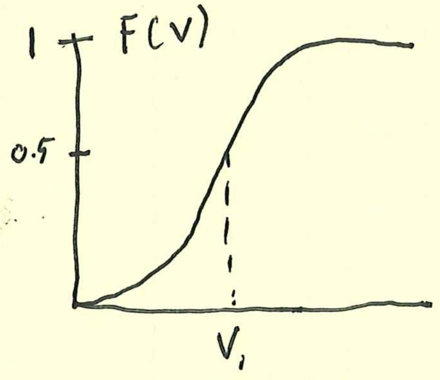
$f_V(v) = \int_0^V f(v)dv$ ✓

For wind:

$$F(V) = \int_0^V \frac{\pi v}{2\langle v\rangle^2} e^{\left[-\pi/4\left(\frac{v}{\langle v\rangle}\right)^2\right]} = e^{\left[-\frac{\pi}{4}\cdot\left(\frac{v}{\langle v\rangle}\right)^2\right]}\Big|_0^V$$

$$\underline{F(V) = 1 - e^{\left[-\pi/4\left(V/\langle v\rangle\right)^2\right]}}$$





half the wind is less than V_1

Also useful: $\text{prob}(v \ge V) = 1 - \text{prob}(v < V) = 1 - F(V)$

for Wind: $\text{prob}(v \ge V) = e^{\left[\pi/4\left(v/\langle v\rangle\right)^2\right]}$

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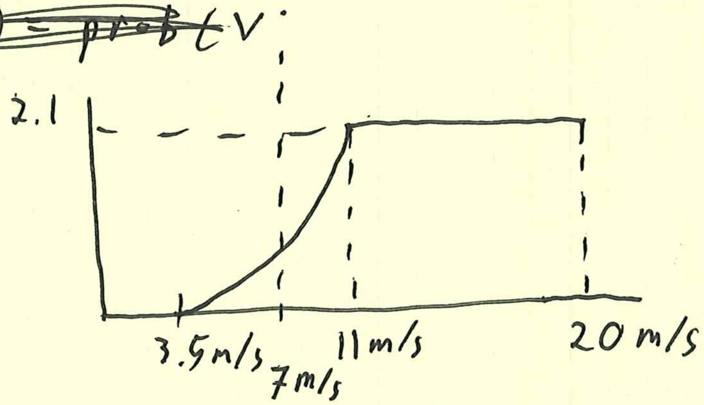
Example (7.7 in Masters's)

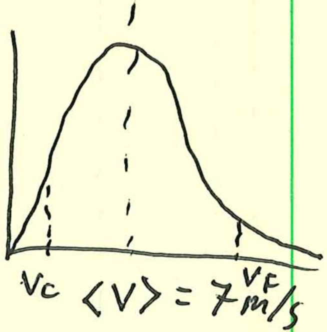
A 2.1 MW wind turbine has a cut-in wind speed $V_c = 3.5$ m/s, rated wind speed, $V_R = 11$ m/s, and a furling speed of $V_F = 20$ m/s. If this machine is located in Rayleigh winds with an average wind speed of 7 m/s, find the following:

a) # of hours/year the wind is below the cut-in speed

~~F(Vc) = prob~~ (V

~~prob~~





$$\text{prob}(V \leq V_c) = F(V_c) = 1 - e\left[-\frac{\pi}{4}\left(\frac{V_c}{\langle V\rangle}\right)^2\right] = 0.178$$

Hours => 0.178×8760 h/yr = 1562 h/yr ~ 2 months

b.) # of hours/year the turbine is shut down due to excessive wind speeds?

$$\text{prob}(V \geq V_F) = 1 - F(V_F) = \exp\left[-\frac{\pi}{4}\left(\frac{V_F}{V_c}\right)^2\right] = 0.00164$$

Hour = 0.00164×8760 = 14.4 hours/yr

c) How many kWhr/yr will be generated when the machine is running at rated power?

$8760 \times \text{Prob.}(V_R \leq V \leq V_F) = \text{Hours}(V \geq V_R) - \underline{\text{Hours}(V \geq V_F)}$

have this

$\text{Hours}(V \geq V_R) = 8760 \cdot (1 - F(V_R))$

$= 8760 \cdot e\left[-\pi/4 \left(11/7\right)^2\right] = 1260 \text{ h/yr}$

$\text{Hours} = 1260 - 14.4 = 1245.4 \text{ h/yr}$

$\text{Energy} = 2100 \text{ KW} \times 1245.4 \frac{\text{h}}{\text{yr}} = \underline{2.62 \times 10^6 \text{ kwh/yr}} = AE_R$

d.) If this turbine in these winds has an overall CF of 38%, what fraction of it's annual energy delivered will be provided by winds below the 11 m/s wind speed?

$AE = PR \times 8760 \times CF = 2100 \text{ Kw} \times 8760 \times 0.38 = 6.99 \times 10^6 \frac{\text{kwh}}{\text{yr}}$

$\% \text{ Energy} \leq 11 = \frac{AE - AE_R}{A_E} = \frac{6.99 - 2.62}{6.99}$

$= 0.625 = 62.5\%$

- Makes sense as the avg wind speed is less than 11 m/s