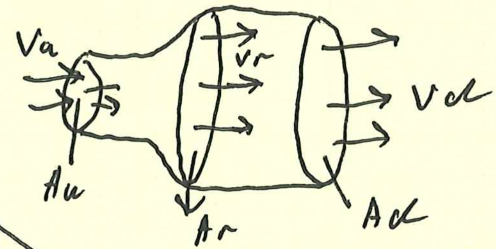
Last time

Betz's limit *

$P_r = \frac{16}{27} \cdot \frac{1}{2} \rho A_r V_u^3$

@ Betz limit $\lambda = \frac{V_d}{V_u} = \frac{1}{3}$



Bernoulli's ~~principle~~ equation

$\frac{1}{2} V^2 + gz + \frac{P}{\rho} = \text{constant}$

* useful for HW 3

Today

Feedback discussion

Rotor Efficiency Curves

~~[illegible]~~ Power Curves

WEIBull + Rayleigh Wind distributions

Feedback

Pace - about right

Hw - mostly useful
wider range on reading

conflicting feedback

↳ some like slides, some like blackboard

↳ hand writing - working on it - pace of talk vs. writing

↳ Didn't like long video

↳ hand notes online - Nope

↳ Piazza - in work

↳ Like the break

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Tip Speed Ratio

Path = πD Tip speed = $\frac{rev}{s} \cdot \pi D$

$$RPM = \frac{rev}{min} \cdot \frac{\cancel{1 min}}{60 s} = \frac{rev}{s} \pi D$$

$$\text{Tip speed} = \frac{\frac{rpm \times \pi D}{60} \ (m/s)}{V \ (m/s)} = \frac{RPM \times \pi D}{60 V}$$

Wind statistics

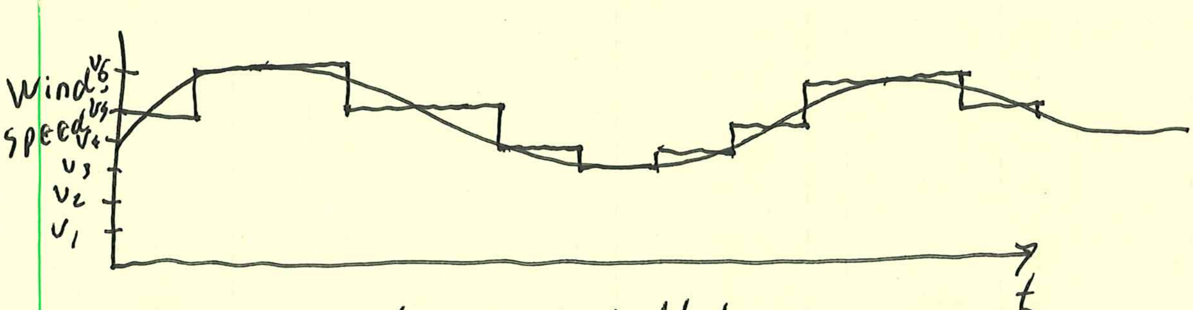
We know we cannot just use the average wind speed of a given site to compute energy

$$P_{avg} = \left(\frac{1}{2} \rho A V^3\right)_{avg} = \frac{1}{2} \rho A (V^3)_{avg} \neq \frac{1}{2} \rho A (V_{avg})^3$$

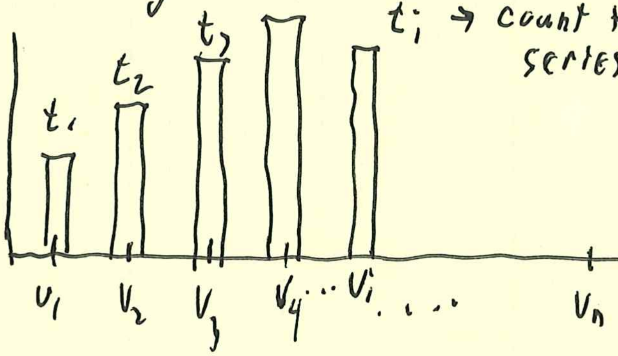
The relation between power & wind speed is non linear.

<u>Discrete Wind Histogram</u>

Given:



we can generate a wind Histogram



t_i → count the # of hours in the time series that the wind blows at speed V_i

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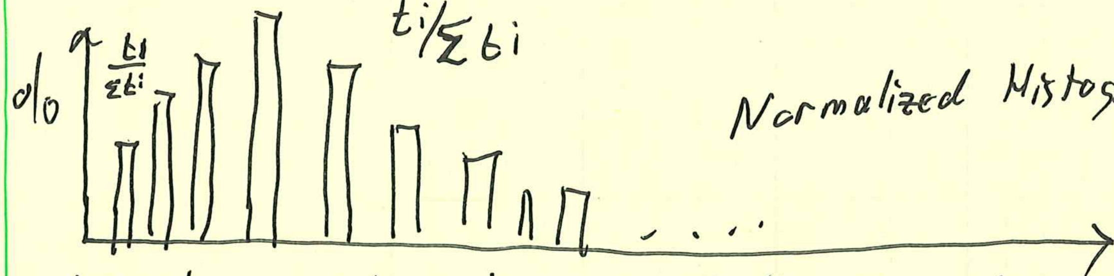
If we normalize the # of hours, divide by the total # of hours we get



All the quantities above are smaller than 1

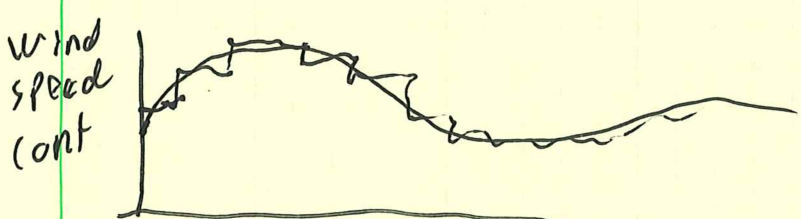
$\frac{t_i}{\Sigma t_i} < 1$ and their sum is exactly equal to one

$$\sum_{i=1}^{n} \frac{t_i}{\Sigma t_i} = 1$$

Since $\frac{t_i}{\Sigma t_i} \geq 0$, it means that the normalized histogram is a proper <u>probability mass function</u> (~~PMF~~) pmf (ECE 313)

<u>Probability Density Function</u>

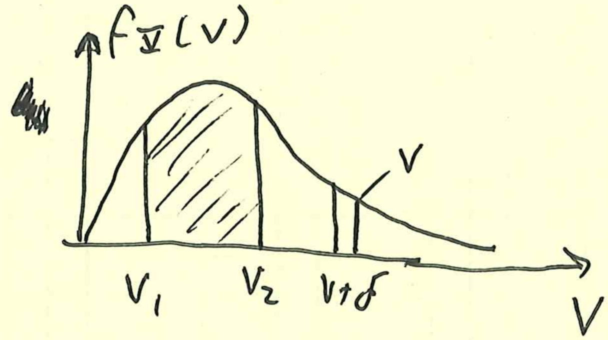
In reality, wind speed does not take discrete values, but it can take on continuous values



=> it is appropriate to think of wind speed as a continuous Random variable => instead of PMF we talk about probability density function (~~PDF~~ pdf)

Let V be a continuous random variable. Its pdf is described by $f_V(v)$ with following properties:

- $f_V(v) \geq 0 \quad \forall V$
- $\int_{-\infty}^{\infty} f_V(v)\,dv = 1$



Prob that $v_1 < V < v_2$
= area under curve
$= \int_{v_1}^{v_2} f_V(v)\,dv$

~~Recall that~~ If δ is small then
Prob that $v < V < v + \delta \simeq f_V(v) \cdot \delta$

The calculation of the average V_{avg} is just the calc. of the expectation of V:

$$V_{avg} = E[V] = \int_{-\infty}^{\infty} v \cdot f_V(v)\,dv$$

we know wind speed ≥ 0

$$\Rightarrow V_{avg} = E[V] = \int_{0}^{\infty} v \cdot f_V(v)\,dv$$

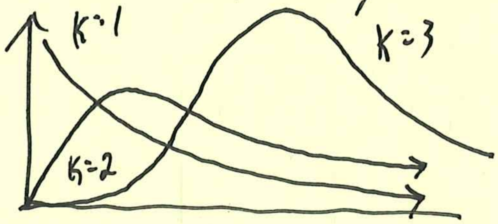
What should we choose for $f_V(v)$?

A popular distribution used to model windspeed is the Weibull distribution, which has a pdf of the form:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \cdot e^{-(v/c)^k}$$

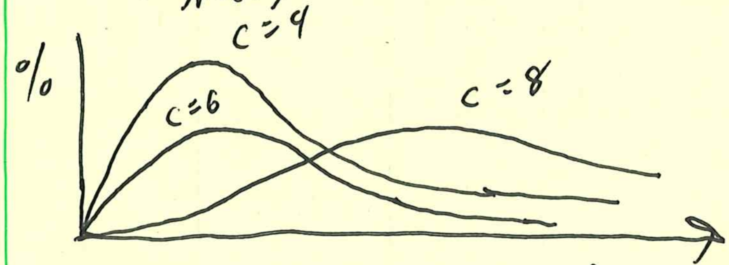
where k is called the shape parameter and c the scale factor.

For $k=1$, we get the exponential function



For $k=2$, the Weibull has a special name: the Rayleigh distribution

The scale parameter shifts the curve toward higher wind speeds



$$f_V(v) = \frac{2}{c}\left(\frac{v}{c}\right) e^{-(v/c)^2}$$

$$E[V] = \int_0^\infty v \cdot \frac{2}{c}\left(\frac{v}{c}\right) e^{-(v/c)^2} dv = \frac{\sqrt{\pi}}{2} c = \langle V \rangle = V_{avg}$$

or $c = \frac{2}{\sqrt{\pi}} \langle V \rangle$

$$f_V(v) = \frac{\pi \cdot v}{2 \langle V \rangle^2} e^{-\frac{\pi}{4}\left(\frac{v}{\langle V \rangle}\right)^2}$$

From ECE 313: Let X be a r.v. with pdf $f_X(x)$

Let $Y = g(X)$ then

$$E[Y] = \langle Y \rangle = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

Fancy way of saying:

$P_W = \frac{1}{2} \rho A V^3$ $\quad (Y = g(X))$

$$\langle P_W \rangle = \int_0^\infty \frac{1}{2} \rho A v^3 \cdot \frac{\pi \cdot v}{2 \langle V \rangle^2} \cdot e^{-\frac{\pi}{4}\left(\frac{v}{\langle V \rangle}\right)^2} dv$$

$$= \frac{6}{\pi}\left(\frac{1}{2} \rho A \langle V \rangle^3\right)$$

$$\boxed{\langle P_W \rangle = \frac{6}{\pi} \frac{1}{2} \rho A \langle V \rangle^3}$$